Adaptive Filter Applications to LIDAR: Return Power and Log Power Estimation

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Abstract—The problem of estimating the return power in a LIDAR system in the presence of multiplicative noise (speckle) is addressed in this paper. A significant advantage of the partitioning approach is applied and comparisons are made with the extended Kalman filter (EKF) in the case where model parameter uncertainty exists. Through extensive simulations, the partitioned filter is shown significantly superior to the EKF algorithm.

I. INTRODUCTION

THE inadequate penetration of infrared and microwave radiation into the water and the passive nature of the visible instrumentation has traditionally hindered the remote sensing of oceanic and atmospheric properties in both active and passive modes. Among alternate observation procedures available, the most viable method is that of obtaining vertical profiles of radar-like range gated systems utilizing lasers as the radiation source. Such systems are referred to as LIDAR [13]–[16]. Atmospheric parameter estimation based on single-pulse LIDAR returns has been investigated in the literature via averaging techniques [3], [12], [19] and methods to cope with the stationarity and correlation properties of the signal have been addressed in detail [11]. In more recent and pioneering work [14], [15], Rye and Hardesty have applied the state-variable formulation and the related Kalman filter and EKF [1], [2], [4], [17] to the estimation of the return power and logarithm of power for incoherent backscatter LIDAR in which multiplicative noise or speckle is present. In this paper, the Rye and Hardesty results are extended to the case of unknown LIDAR state-variable models.

The paper is organized as follows. In Section II, the form of the measurement equation and various possible LIDAR system models in the presence of both additive and multiplicative noise (speckle) are presented with particular reference to estimation of the log power returns. Section III develops the algorithmic basis for the partitioned adaptive filter as parallel implementation of a bank of EKF’s. Section IV reviews the results of extensive simulation studies on the performance of the alternate filters and gives a comprehensive error comparison between the partitioned algorithms and the classical EKF. Section V summarizes significant findings from the study and provides a series of remarks.

II. PROBLEM FORMULATION

A. Measurement Equation

The state to be estimated, \( x_1(k) \), is the signal power corrupted by additive noise, \( v(k) \), which is assumed to have zero mean and Gaussian distribution with covariance \( R(k) \)

\[
Z(k) = \Theta_Z x_1(k) + v(k)
\]

where \( \Theta_Z \) denotes an unknown constant parameter. This is introduced to account for the fact that the designer may have no knowledge of how the states are scaled and appear at the sensor that receives the measurements. To estimate the absorbance of the LIDAR channel, which is proportional to the logarithm of the return power, the measurement equation becomes nonlinear and is given by

\[
Z(k) = \Theta_Z \exp[x_1(k)] + v(k)
\]

with \( \Theta_Z \) having the same meaning as above. Moreover, if one considers a source of multiplicative noise, the measurement equation becomes

\[
Z(k) = \Theta_Z x_1(k) w(k) + v(k)
\]

where \( w(k) \) is the multiplicative noise or speckle defined as

\[
w(k) = 1 + w(k)
\]

with \( w(k) \) being a zero mean white Gaussian sequence and \( \Theta_Z \) as before [15]. The final form of the measurement equation for log power return including the model uncertainties becomes

\[
Z(k) = \Theta_Z \exp[x_1(k)][1 + w(k)] + v(k).
\]

B. System Equations

The signal component along with the speckle can formulate a state space representation

\[
x_1(k + 1) = X(k) + \Theta_w w_1(k)
\]

\[
x_2(k + 1) = 1 + w_2(k)
\]

where \( X_2(k) \) is the speckle term and \( w_1(k), w_2(k) \) are independent zero mean white Gaussian noises with covariances \( Q_1(k), Q_2(k) \), respectively. In meteorological measurements the noise covariance \( Q_1(k) \) is unknown, and the purpose of the \( \Theta_w \) term is to effectively introduce such uncertainty. Given the
system equations (6) and (7), and the measurement equation (5), the EKF algorithm [2], [4] can be applied to estimate the signal power and the multiplicative noise (speckle). Any unknown dynamics are to be determined by making the filter adaptive, a topic reviewed next.

III. ADAPTIVE FILTERING AND EXTENDED KALMAN FILTERING

A. Adaptive Estimation: The Partitioning Approach

The adaptive estimation problem considered is specified by the following equations

\[ X(k + 1) = \Phi(k + 1, k; \Theta)X(k) + \Gamma(k; \Theta)w(k) \]  
\[ Z(k + 1) = H(k + 1; \Theta)X(k + 1) + v(k + 1) \]

where \( X(k) \) is the state vector of the system, \( \Phi(k + 1, k; \Theta) \) is the transition matrix, \( \Gamma(k) \) is the standard deviation of the noise term \( w(k) \), \( Z(k) \) is the measurement vector, \( H(k) \) defines the observation matrix in the measurements, and \( v(k) \) is the additive noise that corrupts the measurements. The unknown parameters are denoted by the vector \( \Theta \), which is considered to be a random variable with known or assumed a priori density \( p(\Theta/0) = p(\Theta) \). The processes \( w(k) \) and \( v(k) \) are still uncorrelated when conditioned on \( \Theta \), with covariances \( Q(k, \Theta) \) and \( R(k; \Theta) \), respectively. Given the measurement set contained in \( Z(k) \), the optimal minimum-mean-square error (MMSE) estimate \( \hat{X}(k/k) \) of \( X(k) \) and the corresponding error covariance are given by [5]-[10]

\[ \hat{X}(k/k) = \int_{\Theta} \hat{X}(k/k; \Theta)p(\Theta/k)d\Theta \]  
\[ P(k/k) = \int \{ (P(k/k; \Theta) + [\hat{X}(k/k) - \hat{X}(k/k; \Theta)] [\hat{X}(k/k) - \hat{X}(k/k; \Theta)]^T \} p(\Theta/k)d\Theta \]

where \( \hat{X}(k/k; \Theta) \) and \( P(k/k; \Theta) \) are the \( \Theta \)-conditional MSE state estimate and the corresponding \( \Theta \)-conditional error covariance matrix. They are obtained from the corresponding linear filter matched to the model with parameter value \( \Theta \) and initialized with the conditions \( X(0/0; \Theta) \) and \( P(0/0; \Theta) \). The sample space of \( \Theta \) is denoted by \( \Omega \) and the value for \( p(\Theta/k) \) is given by

\[ p(\Theta/k) = \frac{L(k/k; \Theta)}{\int_{\Theta} L(k/k; \Theta)p(\Theta/k-1)d\Theta}p(\Theta/k-1) \]

as dictated by Bayes rule formula; the term \( L(k/k; \Theta) \) is the likelihood ratio given by

\[ L(k/k; \Theta) = |Pz(k/k - 1; \Theta)|^{-1/2} \exp[-0.5\|z(k/k - 1; \Theta)\|^2] \]

and \( Pz(k/k - 1; \Theta) \) is the \( \Theta \)-conditional measurement error covariance matrix with \( z(k/k - 1; \Theta) \) being the \( \Theta \) conditional innovation sequence.

Comments:
- The pdf associated with \( \Theta \) is a continuous function of \( \Theta \), and hence a denumerable infinity of linear filters is needed for the exact realization of the optimal estimator.
- The usual approximation is to discretize the sample space \( \Omega \). The integrals in (10)-(12) are replaced by summations running over all possible values of the parameter \( \Theta \).
- It is comforting to know that when the true parameter value lies inside the sample space the adaptive estimator converges to this value. Otherwise, the estimator converges to that value in the sample space that is “closest” to the true value.
- The adaptive estimation problem constitutes a class of nonlinear estimator problems [18]. The partitioning approach decomposes this nonlinear problem into a linear nonadaptive part, consisting of a bank of linear filters, each filter matched to an admissible value of \( \Theta \), and a nonlinear part, consisting of the a posteriori pdf’s \( p(\Theta/k) \) that incorporates the adaptive/learning, or system identifying nature of the estimator.
- An important feature of the partitioning realization of the optimal estimator is its natural decoupled structure. Indeed, all the filters needed to implement the adaptive estimator can be independently realized. As such, the filters can be implemented using parallel processing machines, saving enormous computational time. Also noteworthy is that the overall realization is robust with respect to failure of any of the parallel processors.

Due to the nonlinear nature of the LIDAR model, the optimal adaptive scheme cannot be realized. However, given the linearization that is used in the usual EKF, one may construct a bank of EKF’s in parallel, each one matched to an appropriate value \( \Theta_i \) such that the overall vector \( \Theta = [\Theta_1, \ldots, \Theta_i, \ldots, \Theta_m]^T \) spans the space of the unknown parameters. The partitioned approach can be used to select the EKF conditional model matched to the correct value of the unknown parameter (or the one closest to it). The design is herein referred to as the adaptive Lainiotis extended filter (ALEF).

B. Extended Kalman Filtering (EKF)

The designer of nonlinear filters usually has to resort to some linear approximation, which will enable the use of Kalman filter techniques [4]. One of the simplest approaches is the EKF, in which a Taylor series expansion is used [2]. A prototype of a nonlinear system is described by the following:

System Model

\[ x(k + 1) = f(x(k), k) + w(k) \]

Measurement Model

\[ z(k) = h(x(k), k) + v(k) \]

where \( f(\cdot) \) is a nonlinear function of the state which depends upon the index \( k \), \( w(k) \) is zero mean, white Gaussian noise having variance \( Q(k) \), \( h(\cdot) \) is a nonlinear function of the state which depends upon the index \( k \), and \( v(k) \) is zero mean, white Gaussian noise having variance \( R(k) \). The equations for the pure EKF are well known and are included here only as combined with the partitioned approach. First define a vector \( \Theta \) that contains the model uncertainties (as in Section II)

\[ \Theta = \begin{bmatrix} \Theta_Z \\ \Theta_w \end{bmatrix} \]
The EKF equations are modified to include the vector Θ as given next.

State estimate propagation without any multiplicative noise or speckle (scalar state vector)

\[ X(k+1/k; Θ) = X(k/k; Θ). \]  

State estimate propagation with multiplicative noise (two-dimensional state vector)

\[ \hat{X}(k+1/k; Θ) = \begin{bmatrix} \hat{X}_1(k+1/k; Θ) \\ 1 \end{bmatrix}. \]

Error covariance propagation

\[ P(k+1/k; Θ) = F(k)P(k/k; Θ)F^T(k) + Q(k; Θ). \]

Filter gain

\[ K(k+1; Θ) = P(k+1/k; Θ)H(k+1; Θ) \times \left[ H(k+1; Θ)P(k+1/k; Θ) \times H^T(k; Θ)R(k) \right]^{-1}. \]

State estimate update

\[ \hat{X}(k+1/k+1; Θ) = \hat{X}(k+1/k; Θ) + K(k+1; Θ) \times [Z(k+1) - \hat{h}(x(k+1/k; Θ), k+1/Θ)]. \]

Error covariance update

\[ P(k+1/k+1; Θ) = [I - K(k+1; Θ)H(k+1; Θ)]P(k+1/k; Θ). \]

If the LIDAR problem is treated without multiplicative noise, then the system equations become simply scalar since only one state requires estimation. The following are defined (see Section II):

\[ F(k) = 1 \]
\[ H(k+1; Θ) = Θ_Z \exp(X_1(k+1; Θ)) \]
\[ h(X(k+1; Θ)) = Θ_Z \exp(X_1(k+1; Θ)). \]

The next level of complexity introduces an extra state to account for the speckle returns to be estimated. The problem now becomes two-dimensional, with the first state representing the signal as previously and the second the speckle term. In the latter case, the following definitions apply:

\[ F(k) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \]
\[ H(k+1; Θ) = [Θ_Z X_2(k+1; Θ) \exp(X_1(k+1; Θ)) \times Θ_Z \exp(X_1(k+1; Θ))] \]
\[ h(X(k+1; Θ)) = Θ_Z X_2(k+1; Θ) \exp(X(k+1; Θ)). \]

C. Augmented EKF

An alternative to the partitioning approach is the so-called augmented EKF adaptive scheme. Its adaptation, however, is often slow. Assume first a vector of unknown parameters can be defined:

\[ Θ = \begin{bmatrix} Θ_1 \\ \vdots \\ Θ_N \end{bmatrix}. \]

The state problem of (14) can be augmented to accommodate the unknown vector as follows:

\[ X_a(k+1) = [X_a(k+1)Θ(k+1)]^T \]
\[ X_a(k+1) = \begin{bmatrix} f(X(k)) \\ Θ(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} w(k). \]

Equations (30) and (31) can be used with (15) of the measurements to jointly estimate the unknown parameters of the vector Θ, which is treated as a static noiseless state, and the actual states of interest, which remain as before. The approach suffers when the unknown parameter is a state dependent noise term since the augmentation is not clear to perform.

IV. SIMULATION RESULTS

We now present an overview of the results obtained in the current study. The unknown parameters occur as discussed in Section II, that is, an uncertainty that is noise dependent (and the EKF is simply mismatched) and an uncertainty that appears in the measurements (where the EKF can be augmented and made adaptive). We consider first the case of the unknown noise covariance and then we investigate the uncertainty in the measurements, both for cases of speckle present or speckle absent.

A. Log Power Estimation without Speckle: Unknown Noise Covariance Q_1(k)

Signal sequences containing 200 data points were generated using (6), and observation series using (2). In (2) we set Θ_Z = 1. The signal noise covariance Q_1 is the only unknown, assumed to be varying between 0.0005 and 0.05 uniformly. The uniform distribution corresponds to the worst case scenario to demonstrate no a priori knowledge of the unknown parameter’s probability distribution. The space span for the noise variance Q_1 was chosen based on values from real measurements of LIDAR returns found in [15]. The EKF was used without any inherent mechanism for estimation of the unknown parameters. Since the unknown is a state dependent noise term, the augmented EKF cannot be used. The recursive algorithm starts with initial estimate X(0/0) = 8.1 and initial error covariance P(0/0) = 0.09. The ALEF is designed with three EKF’s in parallel, each one matched to a specific value of Q_1 to cover the entire range of the assumed distribution of Q_1. The approach becomes a multiple model estimator that simultaneously tracks the state trajectories as well as the unknown parameters. The first model sets Q_1 = 0.05, the second with Q_1 = 0.001, and the third with Q_1 = 0.0005. The estimates given by the EKF and ALEF are shown in Fig. 1 along with the true system state. Obviously, the ALEF...
estimator tracks the true trajectory much closer than the EKF. The results of mean square error analysis, over 50 Monte-Carlo runs, are depicted for both filters in Fig. 2. The improvement in the error performance is significant.

B. Log Power Estimation with Speckle: Unknown Noise Covariance $Q_1(k)$

The simulation discussed above is extended to include the multiplicative noise so that the signal measurements are generated using (5) with $\Theta_Z = 1$. To simulate lower order speckle, $x_2(k)$ is generated assuming chi-square statistics of order 14 (see [15]). The speckle is decorrelated between successive measurements and this justifies the approximation given in (6). The measurement equation takes the form of expression (7). All the other terms are similar to those of the previous example and the results for this example are given in Figs. 3–5. The ALEF multimode estimator shows a successful detection of the model that most closely relates to the actual system. In Fig. 6, the histogram (30 bins) of the speckle power estimates is shown to verify the chi-square statistics used in the data generation. The frequency distribution of the filter estimates shows the expected probability density function for the low-order speckle.

C. Log Power Estimation without Speckle: Unknown Matrix $H(k)$

The exact structure of the matrix $H(k)$ in the linearized equation (24), which we denote by $\Theta_Z$, is here unknown and assumed to be taking any of the discrete values 0, 1, 1, and 10. The noise covariance $Q_1(k)$ is now set to 0.001 and held constant. Signal sequences, containing 200 data points were once more generated using (6), and observation series using (2). Adaptive EKF techniques are applied here by augmenting the system to include the unknown parameter as
Fig. 8. Mean square error of the adaptive EKF and ALEF. Unknown $H$.

D. Log Power Estimation with Speckle: Unknown Matrix $H(k)$

The last simulation to be discussed is extended to include the multiplicative noise so that the signal measurements are generated using (5) with fixed $Q_1(k) = 0.001$. To simulate speckle, $X_2(k)$ is again generated assuming chi-square statistics of order 14. All the other terms are similar to those of the previous example. The results for this example are given in Figs. 9 and 10. The performance of the ALEF estimator is shown to be much superior than the adaptive EKF. The reason is the failure of the latter to converge to the correct value of the unknown parameter, which results in the introduction of significant bias.

E. Discussion

It is shown that the partitioning approach as a means of model selection when uncertain environments exist can perform extremely well and far better than conventional estimators for the simple cases of independently unknown time invariant parameters. The performance improvement over mismatched or slowly adaptive schemes is expected to extrapolate itself for more realistic scenarios where the unknowns can be time varying. The degree of approximation needed is dependent upon the time and computational requirements to implement the parallel structures of the partitioning theory.

V. CONCLUSION

Unknown parameters play an important role in the overall estimator performance. The performance of the ALEF is far better than that of the EKF, adaptive or mismatched. The simulations show that the EKF develops a significant bias error from imperfect knowledge of the varying signal noise covariance. The ALEF estimator adjusts to changes in the noise within few time steps as shown in Fig. 4 and eliminates the significant bias error developed by the mismatched EKF. The ALEF multimode estimator also outperforms the augmented EKF version, primarily because of the inability of the latter to accurately follow abrupt changes in the unknown parameters, and the slow adaptation response it exhibits. Due to the highly decoupled structure of the partitioning approach, the computation time of the partitioned ALEF is essentially the same as with the simple EKF. The important results of this work are summarized in Table 1.

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