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Estimating position of mobile terminals from path loss measurements with survey data

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Summary

Estimating the position of mobile terminals is an important problem for cellular networks. A low cost method of locating the mobile terminal is to use measurements of the radio path loss. The distribution of radio path loss is, unfortunately, a non-linear function of the mobile terminal location. The non-linearity results from large obstacles to radio-wave propagation such as buildings or hills. This paper demonstrates how the conditional density of the location given measured path loss can be approximated as a sum of kernel density functions based on radio propagation data collected from propagation surveys or estimated from computer models. Using these approximate density functions an accurate location estimate of a mobile terminal can be estimated from measured path loss values contaminated by measurement noise. Copyright © 2002 John Wiley & Sons, Ltd.

KEY WORDS
robust estimation
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1. Introduction

The market for wireless networking services is undergoing fast growth. This growth is expected to continue with the proliferation of wireless data and digital multimedia devices. Increasingly, individuals are using portable wireless radio devices to access data as well as for voice communication.

A growing concern is the ability to locate individuals making E911 calls with cellular telephones. The FCC in the United States has mandated that cellular network providers must be able to provide an estimated location of terminals making E911 calls that is accurate to within 100 meters for 67 per cent of calls for network-based solutions [1,2].

For proposed third and fourth generation cellular networks, it is envisioned that wireless networks will

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be required to provide higher bandwidth multimedia data with strict Quality of Service requirements [3]. It has been argued that one method to provide these services is to use mobile terminal location and prediction to allocate resources to the terminals [4,5]. Thus, mobile terminal location estimation could well become an integral part of wireless network management systems.

There are other applications such as vehicular fleet management and location sensitive web-browsing which can provide new sources of income for cellular network providers using location estimation technology in the near future [6].

Several methods have been proposed in the literature for the location of mobile terminals in wireless networks based on Angle of Arrival (AoA), Time of Arrival (ToA), or Received Signal Strength (RSS) measurements [7]. Other options explored in the literature include adding GPS receiver hardware to the mobile terminals [8]. GPS can offer very high precision geo-location. This technology has the disadvantages that older mobile terminals cannot be located with this technique and GPS does not work inside buildings or in areas where buildings or hills can block the Line of Sight (LOS) path to the GPS satellites [9].

This paper will discuss the location of the mobile terminals based on the measurement of the propagation path loss between the mobile terminal and the fixed location base stations derived from RSS measurements.

This method has the advantages that it does not require extra measurement hardware in the equipment, does not require strict synchronization between base stations, and can be used with all cellular network configurations with minimal modifications [10].

The propagation path losses between a mobile terminal and fixed base station are clearly functions of the mobile terminal location. If the density functions of the path losses conditional on the location of the mobile terminal were known then an estimate of the location could be easily calculated.

In practice, the conditional densities for the path loss values given the mobile terminal location are not known. The best that can be done is to obtain estimates of median path loss values at fixed locations within the propagation environment.

These values are obtained from either computer models [11], or field surveys of the network area taken during the network planning stage [12].

With this data, there are two approaches that can be taken to construct estimates of the conditional density functions: Parametric and Non-Parametric techniques [13].

Parametric techniques fit some model curves to the survey data and obtain an estimate of the density. The advantage of this technique is that it calculates density functions for locations other than survey point locations. The disadvantage is that the accuracy of these approximate density functions are limited by the degree to which the model matches the actual environment. For 'simple' propagation environments without large obstacles to propagation and with few scatterers, this technique can work very well. For more complex environments such as microcells placed in dense urban environments, parametric models do not work as well [14].

Other work for locating mobile terminals using the RSS derived path loss data have used the parametric modeling technique and then found the Maximum Likelihood Estimate (MLE) of the mobile terminal location [7].

This paper proposes the use of Non-Parametric estimation of the conditional density functions. This technique is a good match to the urban environments which is the region of most interest to cellular network providers.

Work has been performed on using Non-Parametric estimation of measured data densities with Neural Networks [15].

This paper uses the method of creating the Non-Parametric density estimates from sums of kernel density functions. The form of each kernel density function is determined by the sampled data. From this it is possible to generate an estimate of the joint density of the location and measured path loss values with limited knowledge of the conditional density of measured path loss given location.

In the next section, how the approximate conditional density functions are generated will be described. How a minimum variance estimate of the location is calculated from the approximate densities is also detailed. Section 3 will describe the simulations used to evaluate the different location methods. Section 4 will give the results of the application of the estimation methods. Section 5 will summarize our conclusions.

2. Estimation Technique

When a mobile terminal is to be located using path loss measurements, the data available is a set of path loss measurements from k base stations, $\mathbf{Z}$, and a set of survey data for the area that the mobile terminal is known to be residing in. This area is identified by the handoff algorithm of the wireless network. The survey data can be described as a set of true locations, $\theta_1, \theta_2, \ldots, \theta_n$, and path loss vectors $\mathbf{Z}_1, \mathbf{Z}_2, \ldots \mathbf{Z}_n$ where $\mathbf{Z}_j$ is a measured path loss values in decibels taken when a mobile terminal is at location θ_j. The measured path loss values are modeled as

$$\mathbf{Z} = \mathbf{g}(\theta) + \mathbf{V}, \tag{1}$$

where $\mathbf{g}(\theta)$ is a vector function which gives the true median path loss values for locations θ, and $\mathbf{V}_j$ is a zero mean random error vector that is independent of the location θ. The random vector $\mathbf{V}$ models the influence of location independent factors on the path loss such as shadow fading and measurement noise. Equation (1) assumes that the mobile terminal is moving so that short term or fast fading can be averaged out. The length of the $\mathbf{Z}$ and $\mathbf{V}$ vectors is k, the number of base station path loss measurements used to locate the mobile terminal. At least three base station's path loss measurements are needed to get a non ambiguous location estimate. The values in the $\mathbf{Z}_j$ vectors are obtained from either computer models or field survey data.

The location problem can be restated as follows. Given the set of path loss survey data, $\{\mathbf{Z}_1, \mathbf{Z}_2, \ldots \mathbf{Z}_n\}$ and associated true locations $\{\theta_1, \theta_2, \ldots \theta_n\}$, we must find the location $\hat{\theta}$ of a mobile which has a measured path loss vector $\mathbf{Z}$. Note that both the measured path loss vector and the path loss vectors in the survey data set are contaminated with random measurement noise.

The traditional approach to location estimation is to use the Maximum Likelihood Estimator (MLE). The estimated location is calculated using

$$\hat{\theta}_{\mathrm{IMLE}} = \arg\ (\theta) \max f_{\mathbf{Z}|\Theta}(\mathbf{Z}|\theta; \mathbf{P}), \tag{2}$$

where $\mathbf{P}$ is some parametric model of the propagation environment. We will call this estimator the Iterative Maximum Likelihood Estimator (IMLE) since usually iterative techniques are used to solve Equation (2) [16].

An accurate parametric model is unfortunately difficult to obtain for urban environments which are critical for cellular network providers. This makes direct use of Equation (2) problematic. The non-parametric MLE (assuming Gaussian measurement noise density), $\hat{\theta}_{\mathrm{IMLE}}$, can be calculated using [17]

$$\|\mathbf{Z} - \hat{\mathbf{Z}}_j\|^2 = \min \|\mathbf{Z} - \hat{\mathbf{Z}}_i\|^2 \forall i \in \{1, 2, \ldots, n\}$$
$$\rightarrow \hat{\theta}_{\mathrm{MLE}} \approx \theta_j. \tag{3}$$

The approximation improving as $n \rightarrow \infty$. The non-parametric MLE has two shortcomings. First, it can only return estimates of the mobile terminal location equal to the position of one of the survey points. Second, the MLE makes limited use of information from survey points other than the survey point with the measured path loss value closest to the measured signal. To overcome these limitations other non-parametric estimators are proposed.

Assuming the conditional density functions were known, the Minimum Mean Square Error (MMSE) or minimum variance of error estimate for the unknown location θ given the path loss measurement $\mathbf{Z}$ is given by [16,18]:

$$\hat{\theta} = \mathrm{E}\,[\theta|\mathbf{Z}] = \int_{\mathbf{S}} \theta f_{\Theta|\mathbf{Z}}\,(\theta|\mathbf{Z})\ \mathrm{d}\theta$$
$$= \frac{\int_S \theta f_{\Theta,\mathbf{Z}}\,(\theta, \mathbf{Z})\ \mathrm{d}\theta}{\int_S f_{\Theta,\mathbf{Z}}\,(\theta, \mathbf{Z})}\ \mathrm{d}\theta \tag{4}$$

where $\mathbf{S}$ is the area in which the mobile terminal is known to reside.

We will now show the derivation of a non-parametric estimator based on the MMSE in Equation (4). As stated above the densities are not known. The first step is to estimate the joint density as a sum of kernel functions based on the survey data [19]:

$$\hat{f}_{\Theta,\mathbf{Z}}(\theta, \mathbf{z}) = \frac{1}{n} \sum_{j=1}^{n} (h_{\mathbf{z}})^{-d}\,(h_\theta)^{-2}$$
$$\times K_Z\left(\frac{\mathbf{z} - \mathbf{Z}_j}{h_{\mathbf{z}}}\right) K_\theta\left(\frac{\theta - \boldsymbol{\theta}_j}{h_{\boldsymbol{\theta}}}\right). \tag{5}$$

The constants $h_{\mathbf{z}}$ and h_θ are smoothing parameters that determine the width or bandwidth of each of the kernel functions. For simplicity, one usually chooses kernel functions with the properties [13]:

(a) $K(\mathbf{w}) \geq 0\ \forall\ \mathbf{w} \in \mathcal{R}^k$
(b) $\int_{\mathcal{R}^k} K(\mathbf{w})\,\mathrm{d}\mathbf{w} = 1$
(c) $\int_{\mathcal{R}^k} \mathbf{w}K(\mathbf{w})\,\mathrm{d}\mathbf{w} = 0$

where k is the dimension of the kernel. Obviously, the kernel functions are k-variate density functions for random variables of zero mean vectors.

If the estimated density function from Equation (5) is substituted into Equation (4), the non-parametric minimum variance estimator is

$$\hat{\theta} = \frac{\displaystyle\int_{\mathbf{S}} \theta \sum_{j=1}^{n} (h_{\mathbf{z}})^{-k} (h_{\theta})^{-2} \times K_Z \left(\frac{\mathbf{Z} - \mathbf{Z}_j}{h_{\mathbf{z}}}\right) K_{\theta} \left(\frac{\theta - \boldsymbol{\theta}_j}{h_{\boldsymbol{\theta}}}\right) \mathrm{d}\theta}{\displaystyle\int_{\mathbf{S}} \sum_{j=1}^{n} (h_{\mathbf{z}})^{-k} (h_{\theta})^{-2} \times K_Z \left(\frac{\mathbf{Z} - \mathbf{Z}_j}{h_{\mathbf{z}}}\right) K_{\theta} \left(\frac{\theta - \boldsymbol{\theta}_j}{h_{\boldsymbol{\theta}}}\right) \mathrm{d}\theta}. \quad (6)$$

If we assume that $K_{\theta}(\cdot)$ satisfies the properties (b) and (c) above then this simplifies to the expression:

$$\hat{\theta} = \frac{\displaystyle\sum_{j=1}^{n} \theta_j (h_{\mathbf{z}})^{-k} K_Z \left(\frac{\mathbf{Z} - \mathbf{Z}_j}{h_{\mathbf{z}}}\right)}{\displaystyle\sum_{j=1}^{n} (h_{\mathbf{z}})^{-k} K_Z \left(\frac{\mathbf{Z} - \mathbf{Z}_j}{h_{\mathbf{z}}}\right)} = \sum_{j=1}^{n} \theta_j \hat{f}_{\Theta|\mathbf{Z}}(\theta_j|\mathbf{Z}) \quad (7)$$

This creates an estimator of the location using a non-parametric estimate of the density of the path loss values which has the form [19]

$$\hat{\theta} = \sum_{j=1}^{n} \theta_j w(\mathbf{Z}, \mathbf{Z}_j), \quad (8)$$

where n is the number of survey points used, and $w(\mathbf{Z}, \mathbf{Z}_j)$ is a weight function. The weights, written in terms of kernel functions, are defined as:

$$w(\mathbf{Z}, \mathbf{Z}_j) = \frac{K\left(\frac{\mathbf{Z} - \mathbf{Z}_j}{h}\right)}{\displaystyle\sum_{j=1}^{n} K\left(\frac{\mathbf{Z} - \mathbf{Z}_j}{h}\right)} \quad (9)$$

where $K(\cdot)$ is the user selected kernel function for the path loss density, $K_{\mathbf{Z}}(\cdot)$, and h is the smoothing parameter $h_{\mathbf{Z}}$.

The kernel functions used in this paper are listed in Table I. All of these kernels satisfy the requirements (a), (b), and (c) given above. These kernel functions have been shown to provide good density estimates in other similar problem domains [13,19].

A decision critical to the success of this technique is determine the number of survey points to be taken, n. Too small a value for n will result in low accuracy while too large a value of n will result in an expensive survey process with many of the survey points giving little benefit. The Cramer–Rao bound can be used to get a starting estimate of the number of survey points. A reasonable propagation model is assumed and the variance of the location estimate is calculated. An urban LOS path loss model can be taken from Reference [14] for micro-cell environments. (See Appendix A for how the Cramer–Rao bound can be calculated from a path loss model.) The mean standard deviation of the estimated locations has the same order of magnitude as the optimal distance between survey points and an estimate of n can be calculated using this fact. This optimal value of n will be a factor of 2 to 10 times greater

Table I. Kernel functions.

Kernel name	Kernel function $K(\mathbf{x})$	Smoothing parameter (h)
Parzen Gaussian [21]	$\frac{1}{\left(\sqrt{2\pi}\right)^k} \exp\left(-\frac{\|\mathbf{x}\|^2}{2}\right)$	$h = \left[\frac{8k(k+2)(k+4)(2\sqrt{\pi})^k}{(2k+1)C_k}\right]^{\frac{1}{k+4}}$
Parzen Laplace [21]	$\frac{1}{2} \exp\left(-\|\mathbf{x}\|^1\right)$	$h = \left[\frac{8d(k+2)(k+4)(2\sqrt{\pi})^k}{(2k+1)C_k}\right]^{\frac{1}{k+4}}$
Distance based [22]	$\prod_{j=1}^{k} K_p \frac{1}{1 + (\mathbf{x}_j)^p}$	$h = 1.0$
Epanechnikov [13]	$\prod_{j=1}^{k} f(\mathbf{x}_j),\ f(x) = \begin{cases} \frac{3}{4}(1-x^2) & \|x\| \le 1 \\ 0 & \text{otherwise} \end{cases}$	$h = 10\left(\frac{C_k}{n}\right)^{\frac{1}{k+4}}$

$\|\mathbf{x}\|^p$ is the L_p distance of $\mathbf{x}$ from the origin.
C_k is the volume of the hyper-sphere containing $\mathbf{Z}_j$.
$\frac{1}{K_p} = \int_{-\infty}^{\infty} \frac{1}{1+x^p} \mathrm{d}x$.

than the value calculated using the LOS path loss Cramer–Rao bound because of discontinuities in the propagation environment caused by buildings or geographic features which result in Non Line of Sight (NLOS) propagation. The required factor being larger for greater numbers of discontinuities.

The h value controls the amount of smoothing in the density estimate. Larger values of h result in each survey point having a larger region of influence in the sample space for the estimated density. Small values of h mean that the influence region of each survey point is small with the estimated density function becoming a sum of delta functions as $h \to 0$.

The Parzen window kernels require that the volume of the hyper-sphere containing the measurements be estimated. The radius of the hyper-sphere, a, was estimated as being the maximum of the Euclidean distance from the measured value, $\mathbf{Z}$, to each of the survey points being used. The hyper-sphere volume can then be calculated as [13]

$$C_k = \frac{a^k \pi^{\frac{k}{2}}}{\Gamma\left(\frac{k}{2}+1\right)}. \tag{10}$$

The values of h shown for each of the kernels are only the values used in the simulations in this report. The optimal value for each kernel is a function of the actual density function being estimated and are thus unknown for any given estimation problem. It is, however, known that using a value of h that has the correct order of magnitude allows one to obtain results almost as good as using those obtained using the optimal value in many cases [13]. The only general result for the optimal value of h, denoted h^* is that [13]

$$h^* = O\left(n^{-\frac{1}{k+4}}\right), \tag{11}$$

where $a_n = O(b_n)$ specifies that $a_n/b_n \to c$ as $n \to \infty$, where c is some constant value.

In field implementations, real time performance of the algorithm is a major concern. One can take advantage of the property of the kernel functions in Table I that $K(\mathbf{x})$ rapidly goes to zero as $\mathbf{x}$ moves away from the origin. This motivates the optimization that one can speed up the location estimation by only using the N of the total n survey points that have path loss vectors closest to the measured path loss vector $\mathbf{Z}$, as measured by Euclidean distance. All the kernels degenerate to the MLE when $N = 1$.

The optimal value of N is determined by the magnitudes and number of discontinuities in the function $\mathbf{Z}(\theta)$. Lower values of N suffice if $\mathbf{Z}(\theta)$ has several large discontinuities. In practice, the propagation points can be classified into different environments known to have disparate propagation effects. For example, in an urban environment these classifications could be small side street versus main thoroughfare. The number of survey points located in the smallest significant class is often a good choice for N.

3. Description of Simulations

The location estimation methods were evaluated using simulations. A regular Manhattan street micro-cell model was considered with dimensions and propagation characteristics as described in Reference [20]. The environment is shown in Figure 1. The hatched areas represent buildings.

When the Line of Sight (LOS) or shortest distance path between the mobile terminal and base station is unobstructed the median path loss is

$$\bar{z} = 10\log_{10}[d^a(1 + d/g)^b]. \tag{12}$$

The value d is the propagation path length which in the LOS case is the distance between the mobile terminal and base station. When the mobile terminal is in the street and the LOS path is blocked the median path loss is modeled by

$$\bar{z} = 10\log_{10}\{{d_c}^a(1 + d_c/g)^b(d_r)^a[1 + (d_r)/g]^b\}. \tag{13}$$

The radio signal is modeled as diffracting around a corner to reach the mobile terminal. d_c is the distance from the base station to the corner. d_r is the distance from the corner to the mobile terminal.

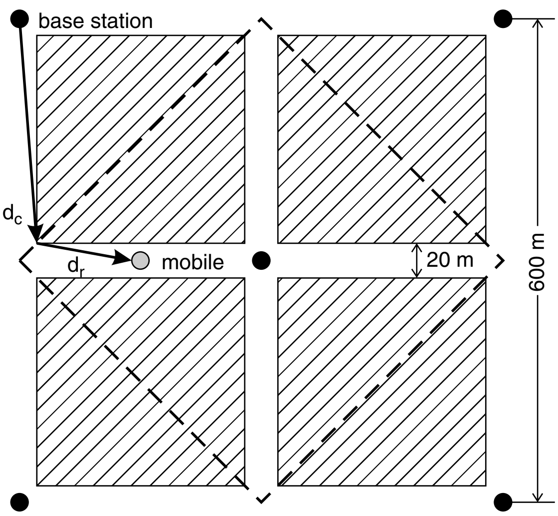

Fig. 1. Manhattan propagation environment.

The values of a and b are constants determining the exponent of the path loss over distance. For the simulations described below a and b are set to be 2 for propagation paths terminating at street locations. g is called the 'breakpoint' distance and is a function of antennae height at the base station and the radio frequency used. The value of g is set to be 150 meters in the simulations below which is typical of urban environments [10].

When the mobile terminal was inside a building the model uses the LOS equation with $a = b = 3$ and adds an additional penalty of 20 dB. This is not intended to be an accurate model of in-building penetration of radio signals. A realistic model would have to take into account the height of the mobile terminal from the ground and building construction materials. The greater path loss for signals propagating into buildings reflects the absorption of radio energy into the walls and the larger amount of radio scattering around the mobile terminal. This simple model is used to see how well the estimation techniques handle the non-linear transitions of the propagation model between different regions. Since Reference [10] does not consider location of mobile terminals inside buildings, an in-building propagation model had to be created. The management of two propagation models by a location estimation system is a realistic scenario a real system would have to face. We consider the cases when the mobile terminal is located only on the street and the case when the mobile terminal can be located either inside a building or on the street when we evaluate the location estimation algorithm.

For comparison, the IMLE estimator from Equation (2) was also implemented. The propagation model for locations inside buildings was used since this model ($(280/300)^2 \approx 87.1$ per cent) is used for most locations.

We assume that the cellular network implements the ideal location based handoff algorithm; the mobile terminal communicates with the base station that is closest to it. The central base station and two other base stations that have the lowest path loss values are used to locate the mobile terminal ($k = 3$). That is, the base station that is serving the mobile terminals and the two other base stations that have the highest measured signal strength from the mobile terminal calculate the radio path loss to the mobile terminal to locate it. This selection method was designed to be consistent with a realistic base station selection method.

Others have assumed that the closest base stations are used [10]. This method was not chosen since it injects an amount of side information into the location estimation process that would not be present in a true implementation of the algorithm. When the closest base stations are used then the presence and exclusion of base stations in the measurement set allows for deterministic removal of areas from S, the area that the mobile terminal is known to be residing in. For example, if we know that base station 1 is closer than base station 2 then all regions in S that are closer to base station 2 than base station 1 can be removed from S. With the path loss based algorithm no such deterministic partitioning is possible.

It is possible with our method of base station selection, theoretically, to calculate a probability for any location in S that the set of base stations making measurements would be selected. These probability values could be used to weight the different survey points and improve the location estimate accuracy. This calculation is not implemented in this paper and is a topic for future research.

A measured path loss value in the simulation is given by

$$\mathbf{Z}(\theta) = \overline{Z}(\theta) + \mathbf{V}, \tag{14}$$

where the entries of $\overline{Z}$ would be calculated using either Equations (12) or (13) depending on the simulated propagation condition. The measurement noises V and V_j, are modeled as zero mean Gaussians with standard deviations of σ dB. This assumption is valid if the mobile terminals are in motion so that the fast fading can be averaged out of the path loss measurements.

The random generated locations for the mobile terminal are all located within the diamond shaped region with dashed boundaries shown in Figure 1. This limited area was selected to avoid edge effects. Two scenarios for locations are explored. The first scenario is when the random location is selected from a random distribution that is uniform over the diamond region.

The propagation environment is dominated by in-building locations. The areas of most interest to cellular network operators, however, are street locations. Since most E911 calls made from cell phones will be made from street locations, evaluation of location error should heavily consider these locations [2].

To see how well the estimators worked for street locations, the location estimation simulations were performed again with the random locations being uniformly distributed only in the street regions of the diamond.

The locations of the survey points are randomly sampled from a uniform distribution over the diamond shaped region. Simulations were performed with a thousand survey points, $n = 1000$, randomly generated from a uniform distribution. This represents the worst case where the estimator has no prior information about the mobile terminals location within the region **S**.

In the field, the survey is likely to be performed in some regular manner with measurement made in some regular pattern. Sets of simulations were performed to evaluate how well the estimators worked when survey points with non-random locations are used. Survey points are placed in a 32-by-32 grid in the diamond region for a total of 1024 points.

The last sets of simulations were performed to evaluate the efficiency of the method for different sizes of survey point sets. The value of n is varied from 100 to 1000 to see how the estimator works with lower densities of survey points. The size of the subset of survey points used for location estimation was set to $N = n/10$.

4. Simulation Results

The figure of merit used to evaluate each estimation method is the Root Mean Squared Error (RMSE) which is the square root of the mean squared distance from the true mobile terminal location and estimated location.

The first set of simulations was run to determine what the optimal value of p, the power of the distance used for the distance based kernel, is for different values of N, the number of survey points used to estimate mobile terminal location. The results are shown in Figure 2. Each point is the RMSE estimated from 10 000 Monte Carlo runs. The results for $N =$ 10, 100, and 1000 have similar minimum values for RMSE obtained when $p = 2$. Based on these results N was set to 10 or 100 for the rest of the simulations with p being set to 2.

The distance based kernel function for $p = 1$ is only valid for cases where the kernel function is windowed so that its value is arbitrarily set to zero for values of **x** that are greater than some distance from the origin. In the simulations this is accomplished by using values of N smaller than n, the total number of survey points. As can be seen from Figure 2, this value gave fairly good results in most cases with a deterioration of performance for $N = 1000 = n$. This is caused by the distance based kernel with $p = 1$ being unable to handle outliers in the survey set for large N. For small N, the outliers are removed before the kernel function is applied so the problem is alleviated.

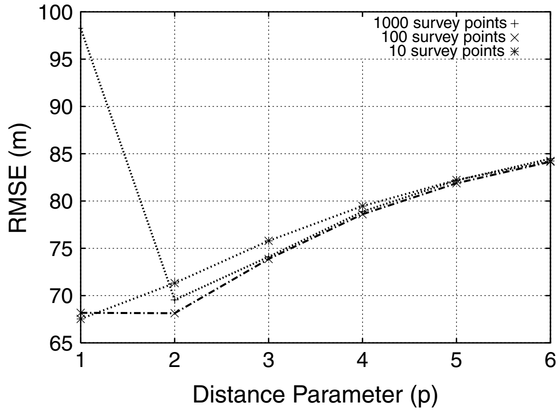

Fig. 2. RMSE vs p value ($\sigma = 4$ dB), distance based kernel.

The optimal value of N depends on the amount of non-linearities caused by Non Line of Sight propagation and diffraction of radio waves. If these phenomena are common, lower values of N should be used. The danger is that if too low a value is selected some of the RMSE reduction of these estimators over the MLE estimator will be lost. A simple technique for determining N is to partition your environment into different classes of regions which should have similar propagation characteristics. For the environment described in this paper a good classification would be street location versus in-building locations. N should be roughly equal to or smaller than the number of survey points in the smallest class of locations. For the environment described in this paper, the smallest class is street locations which are about 12.9 per cent ($\approx$10 per cent) of all locations so $N = 100$ or 10 per cent of the survey points gave fairly good results. The kernel estimators have an 'in-built' ability to reject outliers so picking a value of N slightly high will not have serious consequences.

Simulations were also performed to see how well the estimators would work with different values of h. Figure 3 shows the results for the Parzen window Laplace kernel. As can be seen for $N = 1000$ the value of h calculated in Table I is not optimal and the RMSE is extremely sensitive to variations of h. For the lower values of N, these problems are not evident. The cause of the disparity is that the calculation for h in Table I assumes no discontinuities in the conditional probability $f_{Z|\Theta}(z|\theta)$. If small values of N are used (e.g. 10 or 100) these discontinuities are usually prevented from contaminating the location

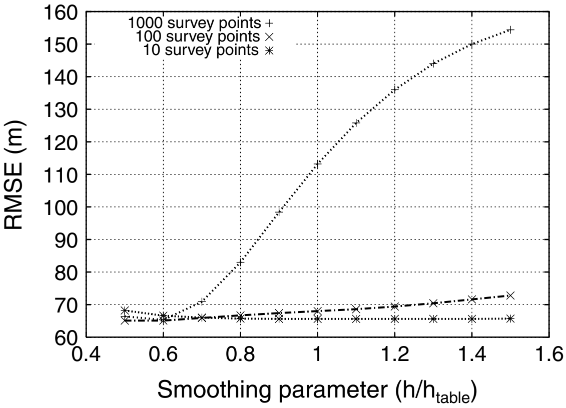

Fig. 3. RMSE vs h value ($\sigma = 4$ dB), Parzen Laplace kernel.

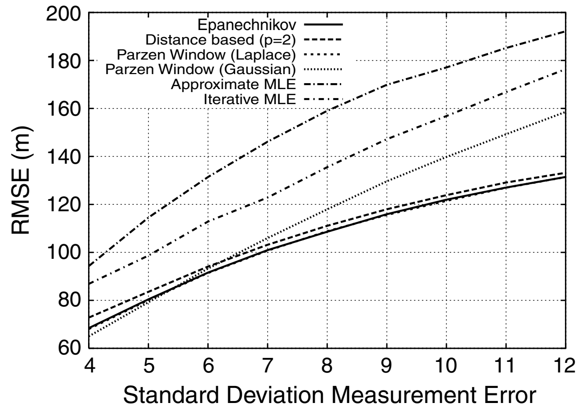

Fig. 4. RMSE of location estimators ($N = 100$).

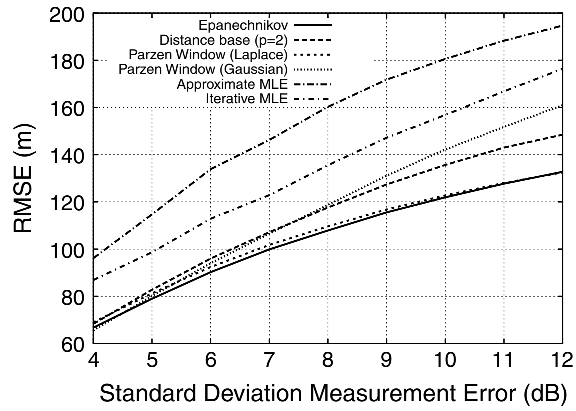

Fig. 5. RMSE of location estimators with deterministic survey point locations ($N = 100$).

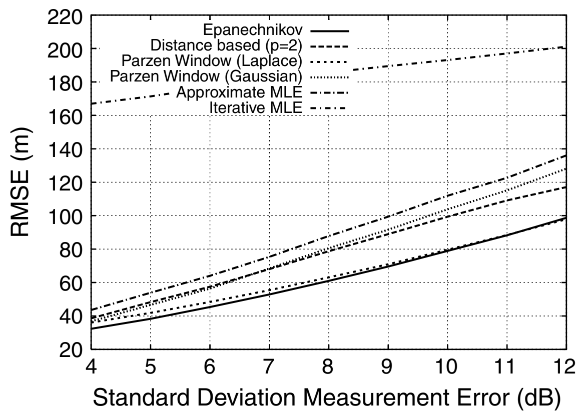

Fig. 6. RMSE for mobile terminals located on street ($N = 100$). Deterministic survey point locations.

estimation process. The results of the Gaussian Parzen window kernel are similar.

The value of p or h for kernel estimators is dependent on the value of N used, the propagation environment, and distribution of the measurement noise. If the selection guidelines for N given above are followed then values of h calculated from Table I give good results. These values assume Gaussian measurement noise which is a fairly reasonable assumption if discontinuities in the conditional probabilities are removed by reasonable selections for N. A rule of thumb is that for low values of N and Gaussian measurement noise a value of $p = 2$ will give good results for the distance based kernel estimator.

Simulation runs were performed to see how robust the different estimators were to variations in the standard deviation of the measurement noise. The results are shown in Figure 4. Each point is the RMSE estimated from 100 000 Monte Carlo runs. A new set of survey points was generated every 100 runs. The approximate MLE gave the highest RMSE with all the kernel estimators giving similar results. The IMLE estimator gave results worse than the kernel estimators but superior to the approximate MLE. The Parzen-like Gaussian window estimator gave worse results than the other kernel-based estimators for higher values of σ.

The simulation results with deterministic survey point locations are shown in Figure 5. The results are similar to the results in Figure 4. The distance based estimator performed slightly worse, showing some sensitivity to survey point locations. From this, it is proposed that the estimators are fairly insensitive to survey point location distribution as long as significant regions are not devoid of survey points.

Simulations were performed for mobile terminals located only at street locations. The results are shown in Figure 6 for $N = 100$, and Figure 7 for $N = 10$. It

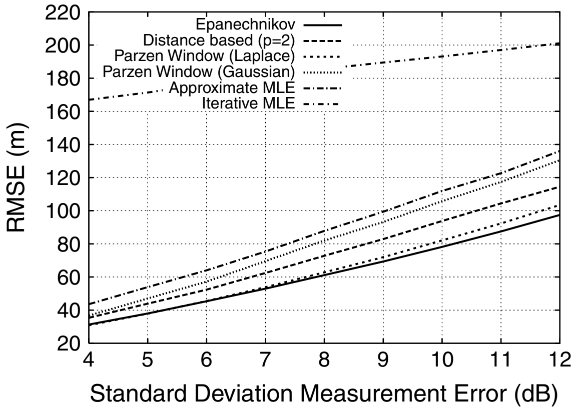

Fig. 7. RMSE for mobile terminals located on street ($N = 10$). Deterministic survey point locations.

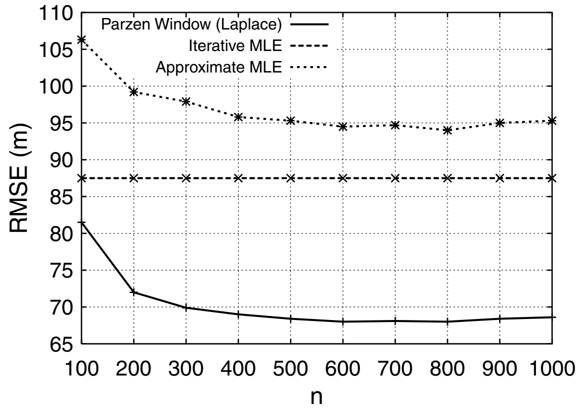

Fig. 8. RMSE for mobile location for different numbers of survey points ($\sigma = 4.0$ dB).

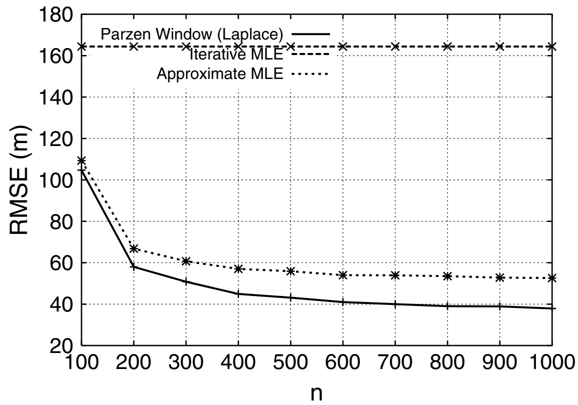

Fig. 9. RMSE for mobile location at street locations for different numbers of survey points ($\sigma = 4.0$ dB).

can be seen that the kernel estimator's RMSE for mobile terminals located on the street is much lower than that calculated for all locations. This is a result of the ability of the non-parametric estimators to use the discontinuity between in-building propagation and on-street propagation. The IMLE's propagation model is optimized for in-building locations so its performance suffers in this case. Note that for the Distance based estimator the results are significantly better for $N = 10$ over $N = 100$.

In all the cases described above, the Epanechnikov and Parzen Laplace kernel estimators gave the best performance. The Epanechnikov kernel has an additional problem over the other kernels. Since its support is finite, it is possible for there to be values of $\mathbf{Z}$ for which Equation (8) results in an attempting to divide 0 by 0 resulting in a worthless result. This occurs when the mobile terminals measured $\mathbf{Z}$ vector is located more than h units away from all the survey points $\mathbf{Z}_j$ measurements for any of the k base stations path loss measurements. The kernel function value will be 0 for all the survey points. This occurred in a maximum of 1 per cent of the random values generated for $\sigma = 12$ dB. For lower values of noise variance the occurrence of the phenomenon was rare, less than 0.1 per cent of cases for $\sigma = 4$ dB.

The last set of simulations is the simulations performed to see how well the estimators handle small survey point sets (i.e. smaller values of n). Once the survey set reaches a certain size, new survey points are adding mostly redundant information. This is evident from the results shown in Figure 8 for mobile terminals with locations distributed over all locations and in Figure 9 for mobile terminals randomly positioned only on street locations. The kernel based estimator performs better than the IMLE estimator for all the cases shown with the gap increasing with the value of n. Increasing n gives reduced rate of returns when $n > 400$. For $n < 100$, not shown in the figures, when the mobile terminal locations are randomly distributed over the whole region, the IMLE has higher accuracy than the non-parametric techniques. The advantage is quickly lost as n increases.

5. Conclusions

The results of this paper show that it is possible to get accurate estimates of the position of mobile terminal location using propagation path loss survey data of the mobile terminal environment. The estimated locations exhibit lower RMSE than the standard MLE location estimator. Methods for calculating good values for the parameters of the kernel functions were presented and the robustness of the method to variations of these parameters from the optimal values were demonstrated. The Laplacian Parzen window kernel gave the best overall results with good robustness to variations in the kernel parameters.

Appendix A

Cramer–Rao Lower Bound for LOS Propagation Location

The Cramer–Rao lower bound is a commonly used technique for obtaining a lower bound on the variance of an unbiased estimator. It does not, by itself, give any information on whether such an estimator exists or how to obtain it. The bound is based upon the inverse of the Fisher information matrix for the conditional density of the measurements given the true value of the parameters to be estimated [16]. For our case, the parameter, θ, is the location of the mobile terminal. The log-likelihood of the measurements given the location with the LOS propagation described in Equation (12) is

$$\begin{aligned} L &= \ln f_{\mathbf{Z}|\Theta}(\mathbf{Z}|\theta) \\ &= \frac{k}{2}\ln(2\pi\sigma^2) - \frac{1}{2\sigma^2} \\ &\quad \times \sum_{j=1}^{k}(z_j - 10\log_{10}({d_j}^a(1+d_j/g)^b))^2 \end{aligned} \quad \text{(A1)}$$

where

$$d_j = \sqrt{(x-x_j)^2 + (y-y_j)^2} \quad \text{(A2)}$$

with (x_j, y_j) being the location of the jth base station. We can then calculate:

$$\begin{aligned} L_{xx} &= \mathrm{E}\left[\frac{\partial^2 L}{\partial x^2}\right] = -\left(\frac{10}{\ln(10)\sigma}\right)^2 \sum_{j=1}^{k} \\ &\quad \times \left[\frac{ag + (a+b)d_j}{d_j^2(g+d_j)}\right]^2 (x-x_j)^2, \end{aligned} \quad \text{(A3)}$$

$$\begin{aligned} L_{yy} &= \mathrm{E}\left[\frac{\partial^2 L}{\partial y^2}\right] = -\left(\frac{10}{\ln(10)\sigma}\right)^2 \sum_{j=1}^{k} \\ &\quad \times \left[\frac{ag + (a+b)d_j}{d_j^2(g+d_j)}\right]^2 (y-y_j)^2, \text{ and} \end{aligned} \quad \text{(A4)}$$

$$\begin{aligned} L_{xy} &= \mathrm{E}\left[\frac{\partial^2 L}{\partial x \partial y}\right] = -\left(\frac{10}{\ln(10)\sigma}\right)^2 \sum_{j=1}^{k} \\ &\quad \times \left[\frac{ag + (a+b)d_j}{d_j^2(g+d_j)}\right]^2 (x-x_j)(y-y_j). \end{aligned} \quad \text{(A5)}$$

The Fisher information matrix is then

$$I(\theta) = \begin{bmatrix} -L_{xx} & -L_{xy} \\ -L_{xy} & -L_{yy} \end{bmatrix}. \quad \text{(A6)}$$

From this the minimum variance of an unbiased estimated position can be calculated from $\mathrm{Var}(\hat{x}) \geq I^{-1}(\theta)_{11}$ and $\mathrm{Var}(\hat{y}) \geq I^{-1}(\theta)_{22}$.

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Michael McGuire is presently a Ph.D. candidate in the Department of Electrical and Computer Engineering at the University of Toronto. He obtained a B.Eng. in Computer Engineering, and an M.A.Sc. from the University of Victoria in 1995 and 1997 respectively. His research interests are estimation and control algorithms for wireless cellular networks.

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