ELSEVIER

Signal Processing 55 (1996) 93–106

SIGNAL PROCESSING

Fuzzy adaptive filters for multichannel image processing

K.N. Plataniotis*, D. Androutsos, A.N. Venetsanopoulos

Department of Electrical and Computer Engineering, University of Toronto, 10 King's College Rd., Toronto, Ont., Canada M5S 3G4

Received 25 January 1995; revised 13 June 1996

Abstract

New filter classes for multichannel image processing are introduced and analyzed. The proposed methodology constitutes a unifying and powerful framework for multichannel image processing. The new filters use fuzzy membership functions based on different distance measures among the image vectors to adapt to local data in the image. The principle behind the new nonlinear filters is explained in detail. Using the proposed methodology multichannel nonlinear problems are treated from a global viewpoint that readily yields and unifies previous, seemingly unrelated results. The new approach provides insight into the nature of the filtering process and the structure of the underlying nonlinear operator. The special case of color image processing is studied as an important example of multichannel image processing. Simulation results indicate that the new filters are computationally attractive and have excellent performance.

Zusammenfassung

Neue Filterklassen für die Mehrkanal-Bildverarbeitung werden eingefürhrt und analysiert. Die vorgeschlagene Vorgehensweise schafft einen einheitlichen, leistungsfähigen Rahmen für die Mehrkanal-Bildverarbeitung. Die neuen Filter verwenden unscharfe Zugehörigkeitsfunktionen auf der Grundlage verschiedener Abstandsmaße zwischen den Bildvektoren zur Anpassung an lokale Bilddaten. Das Prinzip der neuen nichtlinearen Filter wird im einzelnen erklärt. Unter Verwendung der vorgeschlagenen Vorgehensweise werden mehrkanalige nichtlineare Probleme von einem globalen Standpunkt aus behandelt, der unmittelbar frühere, scheinbar nicht zusammenhängende Ergebnisse liefert und vereinheitlicht. Der neue Ansatz liefert Einblicke in die Natur des Filtervorgangs und die Struktur des zugrundeliegenden Operators. Der Sonderfall der Farbbildverarbeitung wird als wichtiges Beispiel mehrkanaliger Bildverarbeitung studiert. Simulationsergebnisse deuten darauf hin, daß die neuen Filter vom Rechenaufwand her attraktiv sind und eine ausgezeichnete Leistungsfähigkeit besitzen.

Résumé

Des classes de filtres originales pour le traitement d'images multi-canaux sont introduites et analysées dans cet article. La méthodologie proposée constitue un cadre puissant d'unification pour le traitement d'images multi-canaux. Les filtres nouveaux utilisent des fonctions d'appartenance floues basées sur différentes mesures de distance parmi les vecteurs d'image afin d'adapter les données locales dans l'image. Le principe sous-jacent des filtres non-linéaires introduits est expliqué en détail. A l'aide de la méthodologie proposée les problémes non-linéaires multi-canaux sont traités d'un point de vue global qui fournit immédiatement et unifie des résultats antérieurs apparemment non liés. L'approche proposée donne une information sur la nature du processus de filtrage et la structure de l'opérateur non-linéaire sous-jacent. Le cas spécial du

*Corresponding author. Tel.: (416) 978-6845, (416) 978-7039; fax: (416) 978-4425; e-mail: kostas@dsp.toronto.edu.

PII S0165-1684(96)00122-3

traitement d'images couleur est étudié en tant qu'exemple important de traitement d'images multi-canaux. Les résultats de simulation indiquent que les filtres introduits sont intéressants du point de vue charge de calcul et ont d'excellentes performances.

Keywords: Adaptive filters; Fuzzy membership functions; Distance functions; Color image processing

1. Introduction

Numerous filtering techniques have been proposed to date for multichannel image processing. The nonlinear filters applied in images are required to preserve edges and details, and remove Gaussian and impulsive noise. One of the most important families of nonlinear image filters is based on order statistics (OS). On the other hand, vector processing of multichannel images is one of the most effective methods to filter and detect edges on multichannel images. Thus, a number of different vector processing filters based on order statistics have been developed lately. This class of filters is very rich. The best known filter is the so-called vector median filter (VMF). The definition of the multichannel median is a direct extension of the corresponding single-channel median definition [9, 10]. However, VMF is not the only member of the OS filter family that is used in multichannel image processing. There exist other order statistic filters (OS) in which the input vector-valued signal at a point is replaced by a linear combination of the ordered vectors in the neighborhood of the point. The class of OS filters includes as special cases the vector median filter, the linear mean filter and the α-trimmed median filter. For a constant signal immersed in additive noise an explicit expression can be derived for the optimal OS filter coefficients [1, 9, 10]. A new class of OS filters for processing vector-valued signals was introduced in [13, 14]. The so-called vector directional filter (VDF) uses the angle between the image vectors as an ordering criterion. The VDF operates on the direction of the image vectors, separating in this way the processing of vector data into *directional processing* and *magnitude processing*.

The performance of the different nonlinear filters based on order statistics depends heavily on the problem under consideration. The types of noise which are present on an image affect the filters' performance. To overcome difficulties associated with the uncertainty about the data, adaptive designs based on local statistics have been introduced [12]. The parameters of the adaptive filter are determined in a data-dependent way. The performance of such filters depends heavily on the accuracy of the estimation of certain signal statistics. A number of test statistics have to be used to estimate the local nature of data. The weights of the adaptive filter are then adjusted according to the values of the test statistics within each processing window. The main problem with a particular adaptive design is that exact statistical analysis is difficult to accomplish and, in general, is time consuming. Another popular adaptive filtering approach is based on the determination of the local nature of the data by appropriate tests applied to the image before the selection of the filter. A number of such decision-based order estimators are reviewed in [9]. Finally, adaptive versions of *L-filters* have been considered recently [6]. It has been found that these adaptive filters have good performance in a variety of different noise characteristics. However, despite their good performance, their main disadvantage is the fact that the original image is needed during training. Unfortunately, in a realistic image processing application this is hardly the case since only the corrupted image is available. Another problem inherent in all the adaptive designs is their computational complexity, especially when compared to the corresponding nonlinear filters based on order statistics [10].

Apart from nonlinear filters based on order statistics a number of fuzzy operators have been applied lately in the field of image processing. Local correlation in the data is utilized by applying the fuzzy rules directly on the pixels which lie within the operational window. The output of the fuzzy processing depends on the fuzzy rule and the defuzzifying process, which combines the effects of the different rules into an output value [11]. However, there is no optimal way to determine the number and the type of fuzzy rules required for the fuzzy image operation. Usually, a large number of rules are necessary. For a medium size window the designer has to compromise between quality and number of rules, since for even a moderate 5×5 window 50 fuzzy rules are required [11].

In this paper a powerful framework is introduced. Fuzzy membership functions based on different distance functions are adopted to determine the weights on a nonlinear adaptive filter. The organization of the paper is as follows. Section 2 presents the general form of the filtering structure. Motivation, and design characteristics are discussed in detail. Section 3 overviews distance criteria used for ordering purposes in image processing. The section also includes the description of the fuzzy weights. Different fuzzy membership functions based on a variety of distance criteria are introduced. Certain classes of adaptive filters are presented in Section 4. In this section relationships with existing filters are also discussed. Applications to color image processing are given in Section 5. Finally, Section 6 summarizes our conclusions.

2. General nonlinear filter framework

2.1. The framework

It was stated in the introduction that several nonlinear filter classes have been used in image processing. Since nonlinear filters originate from different points of view their structure and properties vary widely. The performance of the different nonlinear filters depends heavily on the problem under consideration. Some of these filters are better suited for specific image filtering tasks. The large number of filters available today poses some difficulties to the practitioner, since most of them are designed to perform well in a specific application and their performance deteriorates rapidly under different operation scenarios. Thus, a nonlinear adaptive filter which performs equally well in a wide variety of applications is of great importance. In particular, it is desirable to obtain a unifying nonlinear filter framework, which encompasses the different classes of existing nonlinear filters as special cases. Such a framework is introduced in this section.

The filter structure combines distance concepts with data dependent filters and fuzzy membership functions. The weights of the filter are determined adaptively using fuzzy transformations of a distance criterion at each image position. Thus, the coefficients of the filter are not considered as constants, but are determined in an adaptive way. In this sense, the filter structure is data-dependent. Furthermore, the output of the resulting filter is a nonlinear function of a weighted average of the input values inside the operational window.

Let $y(\boldsymbol{x}) : Z^l \rightarrow Z^m$, represent a multichannel image and let $W \in Z^l$ be a window of finite size n (filter length). The pixels inside the window W will be noted as $\boldsymbol{x}_i,\ i = 1,2,...,n$.

The general form of the new class of filters is given as a nonlinear transformation of a fuzzy weighted average of the input vectors inside the window W:

$$\hat{y} = g\left(\sum_{j=1}^{n} w_j^s \boldsymbol{x}_j\right), \tag{1}$$

$$\hat{y} = g\left(\frac{\sum_{j=1}^{n} w_j \boldsymbol{x}_j}{\sum_{j=1}^{n} w_j}\right), \tag{2}$$

where $g(\cdot)$ is a nonlinear function that operates over the weighted average of the input set. The relationship between the pixel under consideration (window center) and each pixel on the window should be reflected in the decision for the filter's weights. Through the normalization procedure two constraints necessary to ensure that the output is an unbiased estimator are satisfied. Namely:

(i) Each weight is a positive number, $w_j^s \geqslant 0$.

(ii) The summation of all the weights is equal to one, $\sum_{j=1}^{n} w_j^s = 1$.

The fuzzy weights provide the degree to which an input vector contributes to the output of the fuzzy module. According to (1) the output of the fuzzy module corresponds to the fuzzy centroid of the input window [3].

The structure of the general adaptive filter consists of a fuzzy module that delivers an adaptive, fuzzy output and a nonlinear function $g(\cdot)$ which receives as input the fuzzy module and delivers an output which can emulate a number of linear or nonlinear filters.

2.2. Comments

(i) The filter structure described above can support different filter families, depending on the choice of the nonlinear function $g(\cdot)$. The fuzzy module can also be used to provide different filter classes. Depending on the fuzzy membership and distance used, different filters can be devised.

(ii) The proposed structure is nonlinear and adaptive. Nonlinear since it incorporates the function $g(\cdot)$, and adaptive because it utilizes local information to determine weights for the fuzzy module. The utilization of membership functions and the determination of the filter coefficients through membership strengths classify the proposed structure as a fuzzy design [3].

(iii) The proposed framework combines elements from almost all known classes of image filters. Namely, it combines distance functions used in order statistic filters with averaging outputs used in linear filtering and data dependent coefficients used in adaptive filter designs. The nonlinear filters devised from this unifying framework should perform equally well in all image filtering scenarios. In addition, using specific parameter values in the general formula (1) many linear or nonlinear filters can be obtained.

The proposed adaptive nonlinear structure has some similarities with the nonlinear structure proposed in [8] for gray-scale images. The filter structure in [8] consists of a point-wise nonlinear function operating on the data, a network which sorts signals according to their magnitude, multiplication by constant coefficients and a reverse nonlinear point operation. The final nonlinear operation is also present in the general adaptive design presented here. However, there are two fundamental differences. In [8] the choice of the nonlinearities and filter coefficients were based on the types of noise which are assumed present. In the framework proposed here the weights are determined by fuzzy transformations based on features from local data. The fuzzy module extracts information without any a priori knowledge about noise characteristics. In addition, the new structure is based on the concept of *generalized ordering* [15]. The ordering procedure, if activated, is performed in a transform domain and is not applied directly to the input data. A feature extractor, here the fuzzy membership function, provides features that possibly can be ordered. In the sequence, normalization and/or ordering schemes are applied in the transform domain to determine the filters' weights.

At this point the realization of the fuzzy adaptive filters and the associated computational requirements should be discussed. These depend on the specific form of the nonlinear function and the fuzzy module. The computational complexity analysis requires knowledge of the nonlinear function, evaluation of the fuzzy membership functions and the exact form of the ordering process, if any. From a practical standpoint, the proposed framework yields realizations of different filters that may have reduced complexity. The remarkably flexible structure of (1) allows a wide variety of combinations that can meet a number of computational and hardware constraints, especially in real time.

3. The fuzzy module

3.1. Distance functions and multichannel data ordering

In the framework described above, there is no requirement for fuzzy rules or local statistics estimations. Features extracted from local data, here in the form of sum of distances, are used as inputs to the fuzzy weights. Distance criteria have been extensively used to order input data in image processing applications [2, 10]. This is due to the fact that any outliers will be located in the extreme ranks in the sorted data. Consequently, these outliers can be isolated and filtered out. The concept of input ordering, initially applied in scalar quantities, is not easily extended to multivariate data, since there is no universal way to define ordering in multivariate data. However, in the proposed methodology the distance functions are not utilized to order input vectors. Instead, they provide selected features in a reduced space; features used as inputs for the fuzzy membership functions.

A novel distance criterion has been used for ordering purposes in a new class of multichannel filters introduced in [13]. The so-called vector angle criterion orders the input vectors according to the sum of angles with all the other vectors. Let a_i correspond to the input vector $\boldsymbol{x}_i$ defined as

$$a_i = \sum_{j=1}^{n} A(\boldsymbol{x}_i, \boldsymbol{x}_j), \tag{3}$$

where $A(\boldsymbol{x}_i, \boldsymbol{x}_j)$ denotes the angle between the vectors $\boldsymbol{x}_i$, $\boldsymbol{x}_j$ and $0 \leqslant A(\boldsymbol{x}_i, \boldsymbol{x}_j) \leqslant \pi$. If input ordering is required, then an ordering of the $a(i)$'s as $a_{(1)} \leqslant a_{(2)} \leqslant \cdots \leqslant a_{(n)}$ implies the same ordering to the corresponding $\boldsymbol{x}_i$'s: $\boldsymbol{x}_{(1)} \leqslant \boldsymbol{x}_{(2)} \leqslant \cdots \leqslant \boldsymbol{x}_{(n)}$.

Another distance measure used for multichannel vector ordering is the generic L_p metric (*Minkowski metric*), which is defined as follows:

$$d_p(i, j) = \left(\sum_{k=1}^{m} |(x_i^k - x_j^k)|^p \right)^{1/p}, \tag{4}$$

where m is the dimension of the vector $\boldsymbol{x}_i$. Three special cases of the L_p metric are of particular interest. Namely, the *City-block* distance, the *Euclidean distance* and the *Chessboard* distance that correspond to $p = 1, 2, \infty$ respectively. For the window center (vector under consideration) the sum of distances with all the other vectors inside the window is the distance criterion. Let $d_p(i)$ correspond to the vector $\boldsymbol{x}_i$ defined by

$$d_p(i) = \sum_{j=1}^{n} d_p(i, j), \tag{5}$$

where $d_p(i,j)$ is as in (4) and n is the number of input vectors inside the window. If input ordering is required, then an ordering of the $d_p(i)$'s as $d_{p(1)} \leqslant d_{p(2)} \leqslant \cdots \leqslant d_{p(n)}$ implies the same ordering to the corresponding $\boldsymbol{x}_i$'s: $\boldsymbol{x}_{(1)} \leqslant \boldsymbol{x}_{(2)} \leqslant \cdots \leqslant \boldsymbol{x}_{(n)}$.

Finally, in the R-ordering, the *Mahalanobis* distance of a vector $\boldsymbol{x}_j$ from the window center (vector under consideration), $\boldsymbol{x}_i$ is also used. The *Mahalanobis* distance for this case has the general form

$$d_p(i,j) = (\boldsymbol{x}_i - \boldsymbol{x}_j)^{\mathrm{T}} \Sigma^{-1} (\boldsymbol{x}_i - \boldsymbol{x}_j). \tag{6}$$

The matrix Σ can be the identity matrix, the dispersion matrix or the sample dispersion matrix. If you consider the matrix Σ as an identity matrix, the above distance is equivalent to the *Euclidean* distance. Similarly to the definition above we can consider the sum of all the *Mahalanobis* distances as the ordering criterion.

3.2. The fuzzy weights

The weights w_j in (2) are determined using fuzzy membership functions based on distance criteria selected. The fuzzy transformation is not unique. It usually depends on the specific distance measure that is applied to the input data. The different fuzzy functions must meet some desirable characteristics but mainly are required to have a smooth finite output range over the entire input range. Several candidate functions can meet the above specification. Some of them are more suitable than others for a specific distance criterion. We devote our attention to the fuzzy transformations that are suitable for the distance functions discussed above.

If the angle criterion (sum of angles) is used as a distance measure, a sigmoidal membership function is selected. For the vector angle distance criterion, the fuzzy weight w_i has the following form [3]:

$$w_i = \frac{\beta}{1 + \exp(a_i^r)}, \tag{7}$$

where β, r are parameters to be determined. The value of r is used to adjust the weighting effect of the membership function, and β is a weight scale threshold. Since by its definition the vector angle distance criterion delivers a positive number in the interval $[0, n\pi]$, the output of the fuzzy transformation above produces a membership value in the interval $[\beta/(1 + \exp((n\pi)^r)), \beta/2]$. The parameter β, thus far, is chosen arbitrarily. However, limits on a maximum value of β can be imposed. For a 3×3 window with $\beta = 2$, $r = 1$ the output of the membership function lies in the interval $[1.4 \times 10^{-12}, 1]$. Therefore, we can consider the above membership function as having values in the interval $(0, 1]$. Based on the problem under consideration different values of r can be selected. In this way, the filter can produce different output values.

If a generalized norm (L_p metric) is used as distance function, the fuzzy membership function has an exponential form. The following generic fuzzy transformation is used:

$$w_i = \exp[-d_p(i)^r/\beta], \tag{8}$$

where r is a positive constant, and β is a distance threshold. The parameters r and β are design parameters. The actual values of the parameters vary with the application. The above parameters correspond to the denominational and exponential fuzzy generators [3] controlling the amount of *fuzziness* in the fuzzy weight. It is obvious that since the distance measure is always a positive number the output of this fuzzy membership function lies in the interval $[0,1]$. The fuzzy transformation is such that the higher the distance value is, the lower the fuzzy weight becomes. It can easily be seen that the membership function is 1 (maximum value) when the distance value is 0 and 0 (minimum value) when the distance value is infinite.

Alternative fuzzy membership functions can also be used instead of (7) or (8). A simple and efficient one is the inverse of the sum of distances between the pixel under consideration and all the other pixels inside the operational window. This well known fuzzy formula, extensively used for fuzzy clustering and pattern recognition [13], is as follows:

$$w_i = \left(\frac{1}{d_p(i)}\right)^{r/r-1}, \tag{9}$$

where r is positive constant, and $d_p(i)$ is any distance metric from the L_p family. If $p = 2$ then the sum of the *Euclidean* distances inside the operational window is used as input to the fuzzy membership function.

The parameter r is the weighting exponent for the fuzzy transformation. The value of r is used to adjust the weighting effect of the input. When r is large, input vectors with large sum of distances will contribute less. On the other hand, when r is close to 1 the fuzzy weight reduces to inverse of the input value. The value of the parameter r can be determined in an adaptive or on-adaptive way. It has been shown [5] that it is appropriate to choose values for r between 1.25 and 1.75.

It can be seen from the form of (9) that the output of this fuzzy transformation also satisfies the constraints imposed for the weights. Namely, it delivers a positive output and the total value of the normalized weights in (1) is equal to 1. However, the maximum value of the fuzzy strength obtained through this transformation is not one. There is a difference in scale among the fuzzy membership strengths obtained through (7) and (8) and the values generated by (9). The form of (9) provides the means to generate a variety of different fuzzy membership strengths depending on the value of r and the type of the distance function used as input to the fuzzy membership function. If $r = 1$ and $p = 2$, the output of (9) coincides with the heuristic forms of weights used in [4]. It must be emphasized that the *Euclidean norm* is not the only choice. Any particular power of the generic *Minkowski* metric or the normalized inner product can be used as input to (9).

The above definition of fuzzy weights help the filter to transform inputs to outputs without any fuzzy rule. The type of fuzzy transformation and the distance used, determine the outcome of the fuzzy weight and filter. It must be noted that through the fuzzy transformation the size of the operational window actually varies. In smooth image areas the fuzzy membership functions generate equal weights leading to full size filtering. On the contrary, near edges or in areas with high detail, the inputs with the relatively large distance values have smaller weights, leading to a reduction of the filter size.

4. Fuzzy adaptive filters

The general nonlinear filter structure introduced in (1) is versatile and powerful. Its versatility lies in the fact that different families of adaptive filters can be utilized by changing the form of the nonlinear function $g(\cdot)$, as well as the way the fuzzy weights are calculated. By altering the nonlinear function $g(\cdot)$ and/or the fuzzy module, various filters result. The choice of these two design parameters determines the characteristics of the output filter. Three special classes, which constitute fuzzy generalizations of important multichannel filters are studied in the following.

4.1. Fuzzy weighted average filter

The first class of filters devised from the general nonlinear fuzzy algorithm is the so called fuzzy weighted average filters (FWAF). In this case, the nonlinear function g takes the value $g = 1$ and the output of the filter is a fuzzy weighted output of the input set. The form of the filter is given as

$$\hat{y} = \sum_{j=1}^{n} w_j \boldsymbol{x}_j \Big/ \sum_{j=1}^{n} w_j \,. \tag{10}$$

The filter provides a vector-valued signal which is not included in the original set of inputs. The weighted average form of the filter provides a compromise between a nonlinear order statistics filter and an adaptive filter with data dependent coefficients. It is believed that the filter will have good performance in the presence of impulsive noise and also perform well in the presence of short-tailed noise. Notice that depending on the decision of the distance criterion and the corresponding fuzzy transformation, different fuzzy filters can be designed. If the distance criterion selected is the sum of vector angles, the fuzzy vector directional filter (FVDF) is obtained. If an L_1 norm is used as the distance criterion, a fuzzy generalization of the Vector Median Filter (VMF) is obtained.

4.2. Maximum fuzzy vector directional filters

Another form of the nonlinear function $g(\cdot)$ of great importance is the maximum selector. In this case, the output of the nonlinear function is the input vector that corresponds to the maximum fuzzy weight. Using the maximum selector the output of the filter is part of the original input set. The form of the filter is as follows:

$$\hat{y} = \boldsymbol{x}_i, \tag{11}$$

with $w_i^* = \max w_j^*$.

In other words, we select as output the input vector associated with the maximum fuzzy weight. It must be emphasized that through the fuzzy membership function, the maximum fuzzy weight corresponds to the minimum distance. If the vector angle criterion is used to calculate distances, the new fuzzy filter delivers the same output as the basic vector directional filter (BVDF) [13, 14]. If the L_1 norm (*City block* metric) is adopted as distance criterion, the filter provides the same output as the vector median filter (VDF). Utilizing the appropriate distance function different filters can be obtained. Thus, filters, such as VMF or BVDF can be seen as special cases of this specific class of fuzzy filters.

4.3. Fuzzy ordered vector directional filters

It is not necessary for the designer to use all the inputs inside the operational window to produce the final output in the nonlinear filter. If necessary, only a part of the vector-valued input signals can be used. The input vectors are ordered according to their respective fuzzy membership strengths. In the following, only a number of them are used. The form of the fuzzy ordered vector directional filter is given as

$$\hat{y} = \sum_{j=1}^{k} w_{(j)} \boldsymbol{x}_j \Big/ \sum_{j=1}^{k} w_{(j)} \,, \tag{12}$$

where $w_{(j)}$ represents the jth ordered fuzzy membership function, and $w_{(k)} \leqslant w_{(k-1)} \leqslant \cdots \leqslant w_{(1)}$, with $w_{(1)}$ being the fuzzy coefficient with the largest membership strength. The present form of the filter is a special case of the general filter introduced by (1). The nonlinear operator in this case is the ordering of the fuzzy weights. The above form of the algorithm constitutes a fuzzy generalization of the α-trimmed filters [10]. Through the fuzzy transformation, the weights to be sorted are scalar values. In this way the nonlinear ordering process does not introduce any significant computational burden. Depending on the distance criterion and the associate fuzzy form that the designer chooses, a number of different α-trimmed filters can be obtained.

The question which arises is how to select the appropriate number of input vectors that will be included in the final output. There is no standard procedure to determine the number of inputs that are trimmed and not included in the averaging process. There are two different ways to determine the number of vectors that have to be included in the final set that produces the fuzzy output. The first option is the selection of a fixed number of inputs vectors. The filter designer selects k input vectors that correspond to the k fuzzy weights with the largest values. On the other hand, the number of vectors can be determined in an adaptive fashion. The method proposed here is to include all the vectors associated with a fuzzy weight larger than a given threshold. In this work a threshold value of $t = 1/n$, is proposed, with n the number of vectors inside the operational window.

5. Application to color images

Color image processing has become an important area in multichannel image processing. This is mainly due to the numerous real-life applications of color image processing [14]. In this paper, new fuzzy filters from the proposed general classes are evaluated using color test images. Their performance is compared against the performance of the most popular vector processing filters, namely the vector median filter (VMF), and the generalized vector directional filters (GVDF). The general framework described above provides the means to devise a large number of nonlinear fuzzy filters. The designer can use alternative nonlinear functions, different fuzzy transformations and different distances to construct a wide range of different filters. In this paper we devote our attention to the fuzzy weighted average filters of (8). Different forms of the filter can be obtained using alternative distance measures and corresponding fuzzy transformations. Specifically we apply the following fuzzy filters:

- $FVDF^1$ with fuzzy weights of $w_j = 1/(1+\exp(a_j))$ and sum of angles as distance criterion.
- $FVDF^2$ with fuzzy weights of $w_j = 1/(1+\exp(a_j^2))$ and sum of angles as distance criterion.
- FVMF with fuzzy weights of $w_j = \exp[-d_p(j)^{0.5}]$ and the sum of L_1 norms as distance criterion.
- FVMEF with fuzzy weights of $w_j = \exp[-d_p(j)^{0.5}]$ and the sum of L_2 norms as distance criterion.

It must be noted that the fuzzy vector directional filter (FVDF) family is a fuzzy generalization of the well-known family of vector directional filters (VDF). The fuzzy vector median filter (FVMF) is a fuzzy generalization of the vector median filter (VMF) since it uses a fuzzy transformation of the L_1 norm to calculate the fuzzy weights. Similarly, the fuzzy vector mean filter (FVMEF) constitutes a fuzzy generalization of a linear mean filter since it uses the L_2 norm as the distance criterion for the fuzzy membership functions.

The test images selected for the comparison are the color versions of 'Lenna' and 'Lake'. The test images have been contaminated using various noise source models in order to assess the performance of the filters under different noise distributions (Table 1).

A correlation factor $\rho = 0.5$ is used in all the experiments to determine the corruption of pixel (i,j) in a channel, if the same pixel (i,j) is corrupted in any of the other two channels. The normalized mean square error (NMSE) has been used as quantitative measure for evaluation purposes. It is computed as

Table 1
Noise distributions

Number	Noise model
1	Gaussian ($\sigma = 30$)
2	Impulsive (4%)
3	Gaussian ($\sigma = 15$) impulsive (2%)
4	Gaussian ($\sigma = 30$) impulsive (4%)

$$\mathrm{NMSE} = \frac{\sum_{i=0}^{N1}\sum_{j=0}^{N2} \|(y(i,j) - \hat{y}(i,j)\|^2}{\sum_{i=0}^{N1}\sum_{j=0}^{N2} \|(y(i,j)\|^2}, \tag{13}$$

where $N1$, $N2$ are the image dimensions, and $y(i,j)$ and $\hat{y}(i,j)$ denote the original image vector and the estimation at pixel (i,j), respectively. Table 2 summarizes the results obtained for the test image 'Lenna' for a filter window 3×3. The results obtained using a filter window 5×5 are given in Table 3. Similarly, Table 4 summarizes the results obtained for the test image 'Lake' for a filter window 3×3, and the results obtained using a filter window 5×5 are given in Table 5.

In addition to the quantitative evaluation presented above, a qualitative evaluation is necessary since the visual assessment of the processed images is, ultimately, the best subjective measure of the efficiency of any method [13]. Therefore, we present sample processing results in Figs. 1 and 2. Fig. 1(a) shows the color image 'Lenna'. The 'Lenna' image corrupted with (4%) impulsive noise is in Fig. 1(b). Figs. 1(c) and (d) show results of the $FVDF^1$ and FVMEF, respectively. Similarly, Fig. 2(a) shows the color 'Lenna' image corrupted with Gaussian noise ($\sigma = 30$) mixed with (4%) impulsive noise and Figs. 2(b) and (c) show the results of the filtering. Fig. 3(a) shows the color image 'Lake'. In Fig. 3(b) the color 'Lake' image corrupted with Gaussian noise ($\sigma = 15$) mixed with (2%) impulsive noises is depicted. Figs. 3(c) and (d) present the results of the $FVDF^1$ and FVMEF, respectively.

Table 2
NMSE ($\times 10^{-2}$) for the 'Lenna' image, window 3 × 3

Noise model	FVDF[1]	FVDF[2]	FVMF	FVMEF	GVDF	VMF
1	0.7335[a]	0.8935	0.9812	0.9389	1.46	1.60
2	0.2481	0.1981	0.1663	0.1608[a]	0.30	0.33
3	0.401	0.3601[a]	0.3826	0.3656	0.6238	0.5404
4	1.039[a]	1.4063	1.1744	1.1151	1.982	1.6791

[a]Best filter performance.

Table 3
NMSE ($\times 10^{-2}$) for the 'Lenna' image, window 5 × 5

Noise model	FVDF[1]	FVDF[2]	FVMF	FVMEF	GVDF	VMF
1	0.7549	1.2886	0.6718	0.6488[a]	1.08	1.17
2	0.3087	0.3472	0.3040	0.3009[a]	0.3018	0.58
3	0.4076	0.4841	0.4031	0.3938[a]	0.459	0.5172
4	0.9550	1.8115	0.7491	0.7190[a]	1.1044	1.0377

[a]Best filter performance.

Table 4
NMSE ($\times 10^{-2}$) for the 'Lake' image, window 3 × 3

Noise model	FVDF[1]	FVDF[2]	FVMF	FVMEF	GVDF	VMF
1	1.2036[a]	1.7286	1.3659	1.3207	1.52	1.54
2	0.7039	0.718	0.5618	0.5604[a]	0.63	0.66
3	0.7945	0.983	0.7642	0.7490[a]	1.3381	0.9937
4	1.6920	2.9007	1.6014	1.5403[a]	3.053	2.1754

[a]Best filter performance.

Table 5
NMSE ($\times 10^{-2}$) for the 'Lake' image, window 5 × 5

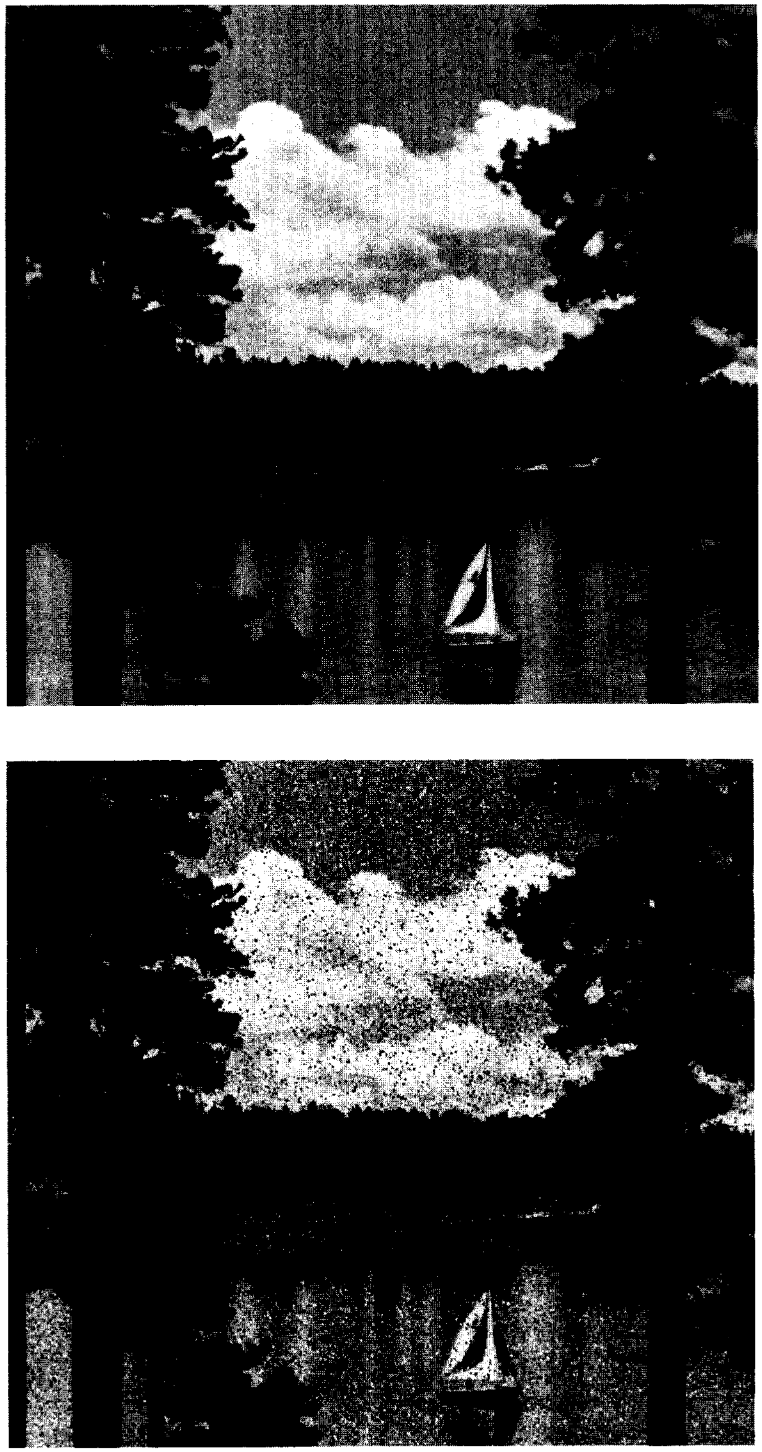
Noise model	FVDF[1]	FVDF[2]	FVMF	FVMEF	GVDF	VMF
1	1.7461	3.046	1.3671	1.3407	1.16[a]	1.1612
2	1.1543	2.447	1.040[a]	1.044	1.3954	1.14
3	1.2615	2.1196	1.1142	1.1079[a]	1.4224	1.2373
4	2.3065	3.1642	1.4583	1.4267[a]	2.3248	1.7804

[a]Best filter performance.

Fig. 1. (a) Color image 'Lenna', (b) 'Lenna' corrupted with (4%) impulsive noise, (c) FDVF[1] of (b) using 3 × 3 *window*, (d) FVMEF of (b) using 3 × 3 *window*.

(a) (b) (c)

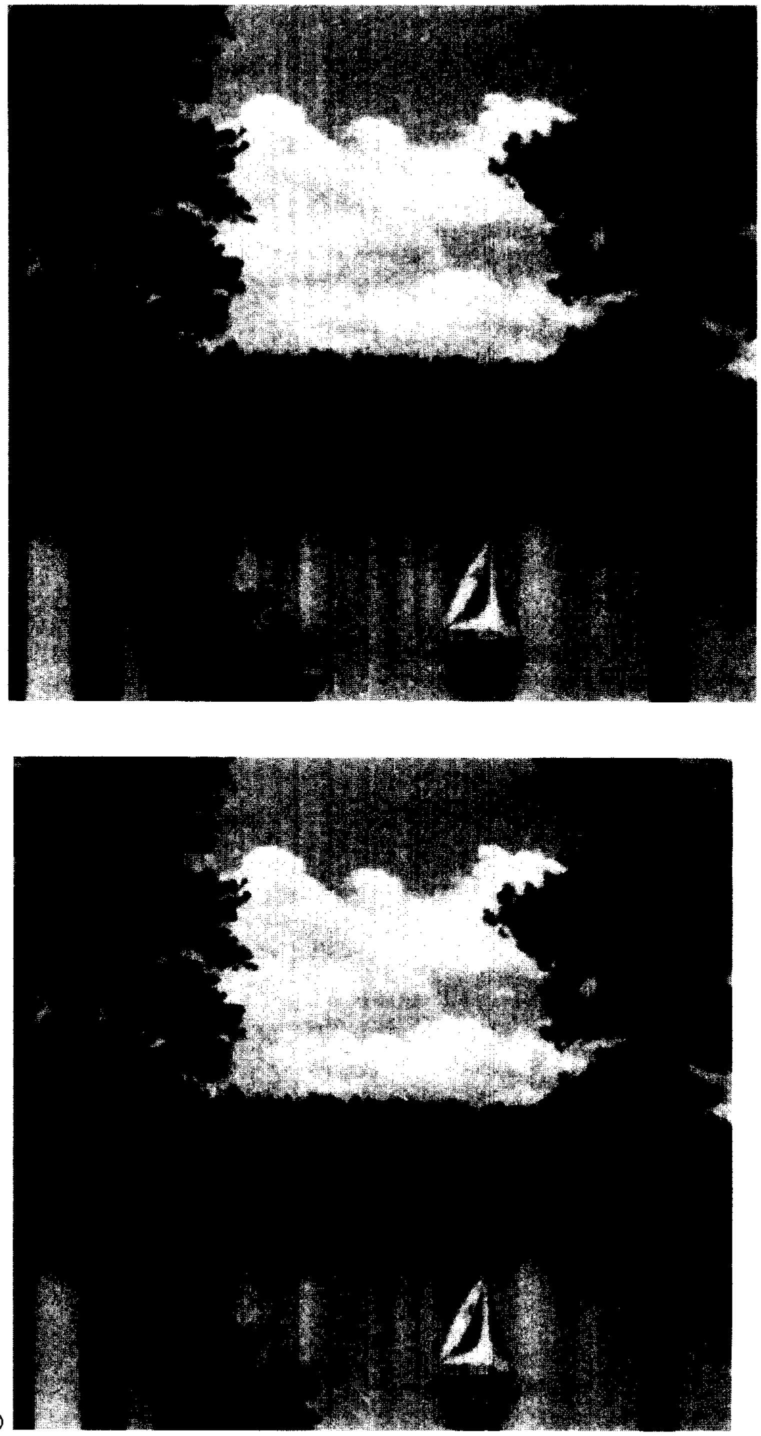
Fig. 2. (a) 'Lenna' corrupted with Gaussian noise ($\sigma = 30$) mixed with (4%) impulsive noise, (b) FDVF[1] of (a) using 3×3 *window*, (c) FVMEF of (a) using 3×3 *window*.



(a)

(b)

Fig. 3. (a) Color image 'Lake', (b) 'Lake' corrupted with Gaussian noise ($\sigma = 15$) mixed with (2%) impulsive noise.



(c)

(d)

Fig. 3. (c) FDVF[1] of (b) using 3 × 3 *window*, (d) FVMEF of (b) using 3 × 3 *window*.

6. Concluding remarks

A general nonlinear filter structure for multichannel image processing was introduced and analyzed in this paper. The new structure combines, in a novel way, nonlinear functions, fuzzy memberships and distance criteria. Several different families of image processing filters can be considered as special cases of this unified general framework. In addition, the general structure introduced here can be used to design new filters which perform well in a variety of different application scenarios. Experimental simulation results have been included to demonstrate the efficiency of the proposed solution. The new filters introduced here outperform all the other nonlinear filters under consideration. Moreover, as it can easily be seen from the attached images, the new filters preserve the chromaticity component which is very important in the visual perception of color images.

7. Notation

$\boldsymbol{x}_i, i = 1,2,\dots,n$: noisy multichannel image vectors
$\hat{\boldsymbol{y}}$: filter output at the window center
w_i, $i = 1,2,\dots,n$: fuzzy weight corresponding to the noisy input vector x_i
w_i^s, $i = 1,2,\dots,n$: normalized fuzzy weight corresponding to the noisy input vector x_i
$w_{(i)}$, $i = 1,2,\dots,n$: the ith ordered fuzzy weight with $w_{(1)} \leqslant w_{(2)} \leqslant \cdots \leqslant w_{(n)}$
$A(\boldsymbol{x}_i, \boldsymbol{x}_j)$: angle between the vectors $\boldsymbol{x}_i, \boldsymbol{x}_j$
a_i, $i = 1,2,\dots,n$: aggregated angle distance associated with the noisy input vector $\boldsymbol{x}_i$
x_i^k with $i = 1,2,\dots,n$ and $k = 1,2,...,m$: the kth component of the $(m \times 1)$ input vector $\boldsymbol{x}_i$
VMF: vector median filter
VDF: vector directional filter
GVDF: generalized vector directional filter
FVDF: fuzzy vector directional filter
FVMF: fuzzy vector median filter
FVMEF: fuzzy vector mean filter

References

[1] J. Astola and Y. Neuvo, "Optimal median type filters for exponential noise distributions", *Signal Processing*, Vol. 17, No. 2, June 1989, pp. 95–104.
[2] M. Barni, V. Cappellini and A. Mecocci, "Fast vector median filter based on Euclidean norm approximation", *IEEE Signal Process. Lett.*, Vol. 1, No. 6, June 1994, pp. 92–94.
[3] J.C. Bezdek and S.K. Pal, eds., *Fuzzy Models for Pattern Recognition*, IEEE Press, New York, 1992.
[4] A. Buchowicz and I. Pitas, "Multichannel distance filters", Proc. *IEEE Internat. Conf. Image Process., ICIP-94*, Vol. II, November 1994, pp. 575–579.
[5] K.P. Chan and Y.S. Cheng, "Clustering of clusters", *Pattern Recognition*, Vol. 25, No. 2, February 1992, pp. 211–217.
[6] C. Cotropoulos and I. Pitas, "Adaptive nonlinear filter for digital signal/image processing", in: C.T. Leondes, ed., *Advances in 2D and 3D Digital Processing (Techniques and Applications)*, Academic Press, New York, 1994, Vol. 67, pp. 263–317.
[7] I. Gath and A.B. Geva, "Unsupervised optimal fuzzy clustering", *IEEE Trans. Pattern Anal. Mach. Intell.*, Vol. 11, No. 7, July 1989, pp. 773–781.
[8] I. Pitas and A.N. Venetsanopoulos, "Nonlinear order statistic filters for image filtering and edge detection", *Signal Processing*, Vol. 10, No. 4, June 1986, pp. 395–413.
[9] I. Pitas and A.N. Venetsanopoulos, *Nonlinear Digital Filters: Principles and Applications*, Kluwer Academic Publishers, Norwell, MA, 1990.
[10] I. Pitas and A.N. Venetsanopoulos, "Order statistics in digital image processing", *Proc. IEEE*, Vol. 80, No. 12, December 1992, pp. 1893–1923.
[11] F. Russo and G. Ramponi, "Nonlinear fuzzy operators for image processing", *Signal Processing*, Vol. 38, No. 3, August 1994, pp. 429–440.
[12] X.Z. Sun and A.N. Venetsanopoulos, "Adaptive schemes for noise filtering and edge detection by use of local statistics", *IEEE Trans. Circuit Systems*, Vol. 35, No. 1, January 1988, pp. 59–69.
[13] P.E. Trahanias and A.N. Venetsanopoulos, "Vector directional filters: A new class of multichannel image processing filters", *IEEE Trans. Image Process.*, Vol. 2, No. 4, October 1993, pp. 528–534.
[14] P.E. Trahanias, I. Pitas and A.N. Venetsanopoulos, "Color image processing", in: C.T. Leondes, ed., *Advances in 2D and 3D Digital Processing (Techniques and Applications)*, Academic Press, New York, Vol. 67, 1994, pp. 45–91.
[15] A.N. Venetsanopoulos, "Digital image processing and analysis", in: *NATO ASI Series in Signal Processing*, Vol. II, North-Holland, Amsterdam, 1987, pp. 575–668.