Sharpening vector median filters

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Abstract

A sharpening vector median (VM) filter for simultaneous denoising and enhancing vector-valued signals is introduced. This filter uses the trimmed aggregated distance minimization concept and robust vector order statistics to enhance edges and image details while retaining the noise removal characteristics of the standard VM operator. The procedure accommodates various design, implementation and application objectives by enhancing the vector-valued signals depending on the local image statistics and/or the user’s needs. The filter properties discussed in this paper are proven and suggest that the proposed solution is a robust vector processing operator. The performance and efficiency of the filter are analyzed and commented upon. Examples from its application to color image filtering and virtual restoration of artworks are provided.

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1. Introduction

Processing color images is a non-trivial extension of gray-scale operations due to the multichannel nature of color images [1]. Since natural red-green-blue (RGB) images usually exhibit significant spectral correlation modern color image processing solutions use vector algebra and vector fields to process color pixels as vectors. In this way, they offer a better match to the true color image characteristics compared to the earlier component-wise or scalar operators which process each color channel separately, thus introducing color shifts and artifacts [1,2].

The majority of vector processing solutions proposed in the last two decades follow more or less the rationale behind the popular vector median (VM) filter [3]. The VM filter has been considered the golden standard of performance in color image processing due to its simplicity, robustness and excellent impulsive noise suppression ability. It is well-known that the output of the VM filter is the image vector which minimizes the aggregated Euclidean ($L_2$ norm) or absolute ($L_1$ norm) distance to other vectors inside the filter window. To reduce the number of operations needed for its implementation, fast VM filtering algorithms were introduced [3,4]. VM variants which take into consideration the...
properties of color spaces were also proposed [5]. Further to that, a number of attempts, for example those in [6–8], were made recently to improve the edge-preserving ability of the VM filter whereas a VM-like filter capable of suppressing additive Gaussian noise as well as mixed noise was introduced in [9]. A detailed overview of vector filters can be found in [10].

However, to the best authors’ knowledge, none of the known vector filtering solutions is capable of sharpening the edges present in the image and/or enhancing the image details. Images are often blurred as a result of filtering. To reduce these effects, we will introduce a unique vector processing operator, called hereafter a sharpening VM filter, which naturally enhances edges in vector-valued signals while still holding robust noise removal ability of the VM filter. The proposed sharpening VM filter is presented in Section 2. Motivation and design characteristics are discussed in detail and the filter is analyzed with respect to their properties and parameters used. Section 3 presents simulation results on the noise removal and edge enhancement of both the standard and proposed here sharpening VM filters. An application to color image filtering and virtual restoration of artworks is shown and the computational complexity of the sharpening VM filter is analyzed in Section 4. Conclusions are offered in Section 5.

2. Sharpening VM filters

Consider a $K_1 \times K_2$ RGB image $\mathbf{x} : \mathbb{Z}^2 \to \mathbb{Z}^3$ representing a two-dimensional (2D) array of three-component vectors $\mathbf{x}_{(r,s)} = [x_{(r,s)1}, x_{(r,s)2}, x_{(r,s)3}]^\top$ occupying the spatial location $(r, s)$, with the row and column indices $r = 1, 2, \ldots, K_1$ and $s = 1, 2, \ldots, K_2$, respectively. In the color pixel $\mathbf{x}_{(r,s)}$ the $x_{(r,s)k}$ value signifies the R ($k = 1$), G ($k = 2$), and B ($k = 3$) component.

Natural images are non-stationary due to the presence of edges and fine details as well as the noise and blur introduced during the image formation [1]. To isolate small image regions, each of which can be treated as stationary, and reduce processing errors by operating in such a localized area of the input image, the sharpening VM filter uses a supporting window $\Psi_{(r,s)} = \{\mathbf{x}_{(i,j)}; (i,j) \in \zeta\}$. The window slides over the entire image $\mathbf{x}$ and the procedure replaces the vector $\mathbf{x}_{(r,s)}$ located at the center $(r, s)$ of the area of support $\zeta$, e.g. $\zeta = \{(r + \phi, s + \phi); -1 \leq \phi \leq 1, -1 \leq \phi \leq 1\}$ for a $3 \times 3$ window, with the output $\mathbf{y}_{(r,s)} = f(\Psi_{(r,s)})$ of a filter function $f(\cdot)$ operating over the samples populating $\Psi_{(r,s)}$. Repeating the procedure for $r = 1, 2, \ldots, K_1$ and $s = 1, 2, \ldots, K_2$ produces color pixels $\mathbf{y}_{(r,s)}$ of the $K_1 \times K_2$ enhanced image $\mathbf{y}$.

The output of the sharpening VM filter is the vector $\mathbf{x}_{(g,h)} \in \Psi_{(r,s)}$ which minimizes the trimmed aggregated distance to other vectors inside the filter window:

$$\mathbf{y}_{(r,s)} = \arg \min_{\mathbf{x}_{(g,h)} \in \Psi_{(r,s)}} \sum_{m=1}^{\zeta} d_{(m)}^{g,h},$$

where the sharpening parameter $\zeta$, with $1 \leq \zeta \leq |\zeta|$ and $|\zeta|$ denoting window size, is defined as follows:

$$\zeta = \max \{\zeta^*\} \text{ subject to } \sum_{m=1}^{1 \leq \zeta^* \leq |\zeta|} d_{(m)}^{g,h} \leq d_{(1)}^{g,h}.$$

The term $d_{(m)}^{g,h} \in \{d_{(ij)}^{g,h}; (i,j) \in \zeta\}$:

$$d_{(1)}^{g,h} \leq d_{(2)}^{g,h} \leq \cdots \leq d_{(|\zeta|)}^{g,h}$$

is the $m$th largest member from the set of Euclidean distances $d_{(ij)}^{g,h} = \sqrt{\sum_{k=1}^{3} (x_{(i,j)k} - x_{(g,h)k})^2}$ between the vectors $\mathbf{x}_{(i,j)}$ and $\mathbf{x}_{(g,h)}$. The lowest (i.e. $m = 1$) rank in (3) always corresponds to $d_{(1)}^{g,h} = 0$ as $d_{(ij)}^{g,h} = 0$ for $(g,h) = (i,j)$. Other ranks, i.e. $m = 2, 3, \ldots, |\zeta|$, indicate the similarity, in a metric sense, between the vector under consideration $\mathbf{x}_{(g,h)} \in \Psi_{(r,s)}$ and its neighbors $\mathbf{x}_{(i,j)} \in \Psi_{(r,s)}$, for $(g,h) \neq (i,j)$.

The output of the proposed filter is by construction one of the input vectors residing inside $\Psi_{(r,s)}$. This is done in order to guarantee local image feature preservation. The minimization concept in (1) demonstrates the noise removal ability of the sharpening VM filter, as the vectors deviating from their neighbors usually denote the outliers whereas the vector minimizing the aggregated distances is the most representative member of $\Psi_{(r,s)}$ and ensures the proper normalization of the image data [1]. If the minimum value of $\sum_{m=1}^{\zeta} d_{(m)}^{g,h}$ is attained by more than one vector including $\mathbf{x}_{(r,s)}$, then the output is defined as $\mathbf{y}_{(r,s)} = \mathbf{x}_{(r,s)}$ in order to avoid processing errors since $\mathbf{x}_{(r,s)}$ occupies the window center.

The objective function $\sum_{m=1}^{\zeta} d_{(m)}^{g,h}$ in (1) allows for signal-detail preservation and edge sharpening, since trimming the aggregated distance excludes outlying vectors, with respect to the vector under consideration, $\mathbf{x}_{(g,h)}$. Low ranks in (3) identify...
vectors which are similar to \( x_{(g,h)} \). Such vectors usually reside on the same side of an image edge. If \((g,h)\) and \((r,s)\) correspond to the same side of a true image edge, the proposed filter preserves the desired structural content. If \((g,h)\) and \((r,s)\) correspond to different sides of a true image edge, the proposed filter enhances edges and fine details by outputting the vector which minimizes \( \sum_{m=1}^{x} d^{ph}_{(m)} \) for \((g,h) \in \zeta \). Thus, the sharpening VM filter with the proper value of \( x \) can simultaneously perform noise smoothing and edge enhancement.

The concept behind (1) can be used in the derivation of gray-scale image operators. This can be done by replacing the Euclidean distance to the absolute differences \( d^{ph}_{i,j} = |x_{(i,j)} - x_{(g,h)}| \) between the scalar samples \( x_{(i,j)} \) and \( x_{(g,h)} \). Table 1 provides the trimmed absolute differences and corresponding results obtained for the scalar set \( \Psi_{(r,s)} = \{200, 115, 71, 113, 185, 70, 112, 110, 70\} \) spanned by a \( 3 \times 3 \) window. The set contains two outliers (i.e., 200 and 185) and a vertical edge comprised of three pixels (i.e., 70, 71 and 70). As it can be seen in Table 1 and Fig. 1, only the use of \( x = 3 \) or \( x = 2 \) leads to simultaneous noise removal and edge enhancement, as this setting results in the replacement of the outlying central sample \( x_{(r,s)} = 185 \) with the edge pixel \( y_{(r,s)} = 70 \). Using \( 4 \leq x \leq 9 \) in the considered scenario corresponds to the noise removal operation, whereas \( x = 1 \) keeps the window center unchanged.

By tuning the parameter \( x \) in (1), different design, implementation and application objectives can be readily addressed. For example, since \( x = |\zeta| \) corresponds to the use of the complete Euclidean distance set \( \{d^{ph}_{(i,j)} ; (i,j) \in \zeta\} \) in the minimization criterion, Eq. (1) can be rewritten as follows:

\[
y_{(r,s)} = \arg \min_{x_{(g,h)}} \sum_{(i,j) \in \zeta} d^{ph}_{i,j}
\]

which is the definition of the VM filter. Therefore, for \( x = |\zeta| \) cost-effective implementations can use directly (4) instead of (1) to avoid ordering the distances \( d^{ph}_{i,j}; (i,j) \in \zeta \) associated with \( x_{(g,h)} \).

For \( x = 1 \), all the trimmed aggregated distances \( \sum_{m=1}^{x} d^{ph}_{(m)} \) for \((g,h) \in \zeta\), reduce to \( d^{ph}_{(1)} = 0 \) or \( d^{ph}_{i,j} = 0 \), suggesting that there is no better candidate \( x_{(g,h)} \) with \( (g,h) \neq (r,s) \) to estimate the vector located at the window center \((r,s)\) than the vector \( x_{(r,s)} \) itself. Thus, (1) with \( x = 1 \) is equivalent to the identity operation \( y_{(r,s)} = x_{(r,s)} \).

For \( |\zeta| > x > 1 \), the filter in (1) performs simultaneously noise smoothing and edge sharpening. In both flat and edge areas, if \( d^{ph}_{(m)} \) is large for lower ranks \( 1 \leq m \leq (|\zeta| + 1)/2 \), then the vector under consideration \( x_{(g,h)} \) represents an outlier in \( \Psi_{(r,s)} \) and will not be passed to the output due to the large value of the objective function \( \sum_{m=1}^{x} d^{ph}_{(m)} \). However, in most cases large \( d^{ph}_{(m)} \) values obtained in flat areas occupy higher ranks in (3), thus identifying the corresponding neighboring vectors \( x_{(i,j)} \) as outliers.

Fig. 1. Visual interpretation of Table 1: (left) noisy input set, (middle) noise smoothing using \( 4 \leq x \leq 9 \), (right) noise smoothing and edge enhancement using \( x = 2 \) or \( x = 3 \).
In high-frequency areas (edges, fine details), higher ranks in (3) usually correspond to outliers or vectors from other side of an edge. Therefore, the amount of enhancement (see Fig. 2) increases with a reduced $\alpha$ value, as the procedure eliminates more outliers and vectors from the other side of an image edge and minimizes the trimmed aggregated distance $P_{am} = \frac{1}{d_{gh}(m)}$ between vectors that are located at the same side of an edge as the vector under consideration $x_{(g,h)}$, as seen in Table 1.

Since excessive enhancement can result in the so-called edge jitter whereas insufficient enhancement can blur edges and remove fine details, selecting $\alpha$ as suggested in (2) forces (1) to follow the local image statistics and perform enhancement adaptively, thus producing visually pleasing images. Such an adaptive solution is suitable for automated imaging systems and applications such as enhancement of television images, aerial and satellite images, and medical images, where the user interaction to control the image quality is undesired or even prohibited due to the real-time processing constraints and/or amount of data to be processed. However, in applications such as virtual restoration of artworks or old photographs and movies, the user is allowed to set the parameter $\alpha$ or re-apply (1) to the same image $x$ until the specific quality criteria are met. Therefore, it is of paramount importance that the choice of $\alpha$ does not restrict the noise removal ability of the filter. Following standard practice, it will be shown that the filter is capable of removing impulsive noise for $1 < \alpha < |\zeta|$, by proving that the impulse response of the sharpening VM filter is zero.

Similarly to the standard VM filter the proposed filter can operate using 1D, 2D, or even 3D windows (e.g. in spatiotemporal color video processing). For the sake of simplicity let us consider a set of vectors $\{x_{(i,j)}; (i,j) \in \zeta\}$ with $\zeta = (r, s + z); -b \leq z \leq b$ denoting locations inside a 1D window. In this set, the center vector $x_{(r,s)} = v$, i.e. for $z = 0$, denotes an impulse $v = [v_1, v_2, v_3]^T$ with magnitude $\|v\|_2 = \sqrt{\sum_{k=1}^{3} (v_k)^2}$ whereas all the remaining vectors $x_{(i,j+z)} = 0$ for $-b \leq z < 0$ and $0 \leq z \leq b$ are zero vectors with $\theta = [0, 0, 0]^T$. Thus, an arbitrary vector $x_{(i,j)}$ is associated with the distances $d_{(m)}^{h} \in \{d_{ij}^{g}; g = i, -b \leq j \leq b, -b \leq h \leq b\}$ from (3) whose values are expressed as follows:

$$d_{m}^{h} = \begin{cases} 0 & \text{for } m \leq |\zeta| - 1 \text{ if } h \neq s, \\ 0 & \text{for } m = 1 \text{ if } h = s, \\ \|v\|_2 & \text{otherwise.} \end{cases}$$

Fig. 2. Enhancement of cDNA microarray images and gray-scale images: (a) input images; (b) $\alpha = 9$; (c) $\alpha = 8$; (d) $\alpha = 6$; (e) $\alpha = 4$; (f) $\alpha = 2$. 

$\|v\|_2$
The corresponding trimmed aggregated distances

\[
\sum_{m=1}^{2} d_{(m)}^{p,h} = \begin{cases} 
0 & \text{for } x = 1, \\
0 & \text{for } x < |\zeta| \text{ if } h \neq s, \\
\|v\|_2 & \text{for } x = |\zeta| \text{ if } h \neq s, \\
(x - 1)\|v\|_2 & \text{for } x > 1 \text{ if } h = s,
\end{cases}
\]  

(6)
suggest that the impulse response of the sharpening VM filter is zero, as for \(x > 1\) the filter always replaces \(x_{(r,s)} = v\) with \(0\) by outputting a vector which minimizes \(\sum_{m=1}^{2} d_{(m)}^{p,h}\) in (6).

In some applications, such as television image enhancement, an impulse can have varying length if it is comprised of identical or very close outlying vectors. The scenario can be modelled using a vector-valued signal \(\{x_{(i,j)}; (i,j) \in \zeta\}\) with the area of support \(\zeta = \{(r,s+z); -b \leq z \leq b\}\), a length \(b\) impulse \(x_{(r,s+z)} = v\) for \(0 \leq z < b\) (note that \(b = (|\zeta| - 1)/2\) here) and the desired zero vectors \(x_{(r,s+z)} = 0\) at all the remaining locations, i.e. for \(-b \leq z < 0\) and \(z = b\). The above implies the following:

\[
d_{(m)}^{p,h} = \begin{cases} 
0 & \text{for } m \leq b + 1 \text{ if } s - b \leq h < 0 \\
0 & \text{for } m \leq b \text{ if } s \leq h < s + b, \\
\|v\|_2 & \text{otherwise},
\end{cases}
\]  

(7)

\[
\sum_{m=1}^{2} d_{(m)}^{p,h} = \begin{cases} 
0 & \text{for } x \leq b + 1 \text{ if } s - b \leq h < 0 \text{ or } h = s + b, \\
0 & \text{for } x \leq b \text{ if } s \leq h < s + b, \\
(x - b)\|v\|_2 & \text{for } x > b + 1 \text{ if } s - b \leq h < 0 \text{ or } h = s + b, \\
(x - b)\|v\|_2 & \text{for } x > b \text{ if } s \leq h < s + b.
\end{cases}
\]  

(8)

Eq. (8) suggests that for \(x > b\) the filter in (1) removes length \(b\) impulses since the aggregated distances corresponding to \(s - b \leq h < 0\) and \(h = s + b\) are smaller than those obtained for \(s \leq h < s + b\).

In practice, \(\Psi_{(r,s)}\) often contains outliers which vary significantly from each other as well as from the noise-free vectors in \(\Psi_{(r,s)}\). In this case, trimming the aggregated distances significantly reduces the value of the objective function corresponding to noise-free samples compared to the values corresponding to the outliers. Thus, minimizing the objective function in (1) identifies noise-free vectors as the filter output. Fig. 3 shows the concept for the set of five input vectors (i.e. \(|\zeta| = 5\)). The set consists of two outliers including the window center \(x_{(r,s)}\) and the filter replaces \(x_{(r,s)}\) with noise-free vector for any value of \(x\) ranged by \(1 < x \leq |\zeta|\), however, only \(x = 2\) and \(x = 3\) enforce edge enhancement.

The above example suggests that the sharpening VM filter has a break-down point greater than that of the VM filter. Consider that \(\Psi_{(r,s)}\) consists of \(|\zeta| - b\) very close noise-free vectors and \(b < (|\zeta| + 1)/2\) outliers varying in their values. Also consider that the vectors in \(\Psi_{(r,s)}\) are centered in the vector space around \(x_{(j)}\) which is an outlier minimizing the objective function in (4). In this case, the output of the VM filter is an outlier whereas the output of the sharpening VM filter is certainly a noise-free vector if \(1 < x \leq |\zeta| - b\). Based on the previous analysis in (8), similar conclusion can be also reached for \(\Psi_{(r,s)}\) with \(b > (|\zeta| + 1)/2\) identical outliers \(v\) and \(|\zeta| - b\) desired zero vectors including \(x_{(r,s)} = 0\). In this case, the output of the VM filter is an outlier whereas the sharpening VM filter produces the desired vector if \(x < b\).

In addition to noise removal the new filter can preserve edges and fine details. It will be shown that the sharpening VM filter possesses root signals [6] meaning that some signal features are invariant to filtering by (1). Knowing the root signals of a given filter allows for the elimination of extensive calculations in order to design fast filtering algorithms. Note that any input signal is always invariant to filtering by the sharpening VM filter with \(x = 1\), as this setting passes \(x_{(r,s)}\) to the filter output unchanged. Furthermore, it was proven in [3] that the VM filter (special case in (1) with \(x = |\zeta|\)
has root signals. Therefore, it will suffice to show the existence of root signals for the sharpening VM filter when \(1 < \alpha < |\xi|\).

The response of (1) to any input signal is uniquely defined with the output \(y_{(r,s)} \in \Psi_{(r,s)}\). Therefore, the root signal \(x_{(r,s)} = f_2(\Psi_{(r,s)})\) can be obtained by filtering repeatedly with a sharpening VM operator \(f_2(\cdot)\) any finite-length signal. Given the extreme complexity of characterizing the roots in multichannel data sets [3] and due to the localized nature of color image features, we restrict the analysis to basic signal structures [6], such as multichannel constant regions, step edges, and impulses.

A multichannel constant region, which is a neighborhood formed by at least \((|\xi| - 1)/2\) identical image vectors, is a root of (1) with \(1 \leq \alpha \leq |\xi|\) if \(x_{(r,s)}\) is one of these identical vectors. This observation results from the fact that \(x_{(r,s)}\) is one of the vectors which minimize \(\sum_{m=1}^{\alpha} d_{(m)}^{i,h}\) for any \(\alpha\).

Similarly, a multichannel step edge, which is a multichannel constant region \(\Psi_{K}\) of \(K\) vectors followed by another multichannel constant region \(\Psi_{[|\xi| - K]}\) of \(|\xi| - K\) vectors, is a root of the sharpening VM filter. If \(x_{(r,s)} \in \Psi_{K}\) and \(K > |\xi| - K\), then \(x_{(r,s)}\) minimizes the criterion in (1) for \(1 \leq \alpha \leq |\xi|\). If \(x_{(r,s)} \in \Psi_{[|\xi| - K]}\), then \(x_{(r,s)}\) minimizes (1) for \(1 \leq \alpha \leq K\), as seen from (8).

A multichannel impulse, which is an image vector \(x_{(r,s)}\) significantly deviating from surrounding multichannel constant region(s) in \(\Psi_{(r,s)}\), is not a root of sharpening VM filters if \(\alpha > 1\), and therefore it will be removed by the filtering operation. This observation follows the previous analysis in (5) and (6).

Finally, it should be noted that similarly to the VM filter, the sharpening VM filter is invariant to rotation, scale, and bias. The sharpening VM filter operates on the ordered set of Euclidean distances \(d_{(m)}^{i,h} = \|x_{(g,h)} - x_{(i,j)}\|_2\); \((g,h) \in \xi\). Since a rotation of the coordinate system does not affect the Euclidean norm of a vector, the filter proposed in (1) is rotation invariant. Moreover, since altering the vectors \(x_{(r,s)}\) to \(px_{(r,s)} + c\) via a scalar constant \(p\) and a vector constant \(c = [c_1, c_2, c_3]^T\) scales the trimmed aggregated distances to \(p\sum_{m=1}^{\alpha} d_{(m)}^{i,h}\), as \(\|(px_{(g,h)} + c) - (px_{(i,j)} + c)\|_2 = p\|x_{(g,h)} - x_{(i,j)}\|_2\), the sharpening VM filter is also invariant to scale and bias, i.e. it poses the following property:

\[
f_2((px_{(i,j)} + c; (i,j) \in \xi)) = pf_2((x_{(i,j)}; (i,j) \in \xi)) + c,
\]

where \(f_2(\cdot)\) is a filter function.

3. Performance analysis

The performance of the proposed filter is examined using a variety of test color images and real artwork images. Examples are shown in Fig. 4. These images vary in color appearance and in the complexity of the structural content indicated through different density and orientation of edges, fine details and lines.

3.1. Experimentation using test color images

Figs. 5 and 6 depict the results cropped in areas of significant structural content. These results correspond to the suppression of synthetic noise, namely 10% impulsive noise and mixed noise (additive Gaussian noise with \(\sigma = 20\) followed by 5% impulsive noise), respectively. The reader can find the corresponding noise models in [10]. This scenario allows for visual (subjective) comparisons of the original images with the noisy and filtered (output) images in order to evaluate noise removal and edge-enhancement capabilities of the proposed filter.

As can be seen, the sharpening VM filter suppresses impulsive and mixed noise as much as the VM filter when the same supporting window (standard \(3 \times 3\)) is used. The results obtained via the proposed filter using a fixed parameter \(\alpha \in \{7, 5, 3\}\), as well as those obtained using an \(\alpha\) adaptively determined via (2), suggest that the sharpening VM filter is capable of removing noise and impairments.
in color images. Careful examination of the filtered results in Figs. 5(c) and 6(c) reveals that the VM filter blurs edges and removes fine details during filtering. This is not the case of the sharpening VM filter (see Figs. 5(d–g) and 6(d–g)) which enhances edges regardless their density, orientation, length and width. Such performance characteristics can be seen, for example, for diagonal and horizontal edges in the parrot heads and for high contrast edges of various orientation in the fruit image. Maintaining the sharpness of the edges is as important as removing the image noise, because edges provide an indication of the shape of the objects in the image. Therefore, it is of paramount importance that the proposed filter not only removes noise and preserves the color/structural content, but also enhances edges and image details.

The presence of the original test images suggests that the filters under consideration can be evaluated objectively. In this work, we use the mean absolute
error (MAE), the mean square error (MSE), and the normalized color difference (NCD) criterion which are widely used in the image processing community to evaluate, respectively, signal-detail preservation, noise suppression, and color distortion. The reader can find the corresponding definitions in [10]. In the experiment, impulsive noise with the corruption probability ranged from 0% to 70% was used to simulate noise corruption. Fig. 7 summarizes the results obtained by processing the noisy versions of the original images. Inspection of the plots reveals that for low noise probabilities (less than 20%) the VM filter produced better results than the proposed solution. This is due to the fact that edges enhanced using the proposed filter are interpreted by the employed criteria as the error (deviation from the

Fig. 6. Filtering of mixed noise (additive Gaussian noise with $\sigma = 20$ followed by 5% impulsive noise) demonstrated for cropped $42 \times 60$ areas: (a) original images; (b) noisy images; (c) VM filter output; (d–g) sharpening VM filter output with (d) $\alpha = 7$; (e) $\alpha = 5$; (f) $\alpha = 3$; (g) $\alpha$ by (2).
original structural content). Increasing the amount of noise in the image reduces the desired structural information needed to guide the sharpening process. Therefore, for higher noise corruption probabilities (more than 20%), the proposed filter usually performs modest sharpening and achieves trade-off between noise suppression and edge enhancement, thus clearly outperforming the VM filter in terms of all objective criteria. To demonstrate the superiority of the proposed solution over the VM filter, Fig. 8 shows the results obtained by filtering the test images which were blurred by a $3 \times 3$ mean filter prior to the introduction of impulsive noise. In this case, edge enhancement produced by the proposed solution should be understood as the attempt to restore the original structural content from the noisy blurred image. Inspection of the plots suggests that if images are affected by blur, then the proposed solution outperforms the VM filter regardless the amount of impulsive noise present in the image. This confirms that the proposed solution enhances edges and fine details while suppressing noise present in the image.

Finally, Fig. 9 shows the root signals of the considered filters. The proposed filter produces higher visual quality of the roots than the VM filter. As the values listed in the figure caption indicate, the proposed filter has faster convergence to the root signals, suggesting its signal-detail preserving ability.

3.2. Experimentation using real artwork images

Fig. 10 shows the results obtained by processing digitized artwork images. Real images shown in Fig. 10(a) were used as the input. These images are corrupted by real, non-synthetic noise with characteristics and statistical properties qualitatively different from the noise properties assumed in the previous experiment. Since original images are unavailable, only visual comparisons were performed. Visual inspection of the results reveals that also in this case the VM-filtered images (Fig. 10(b)) suffer from the reduced sharpness whereas the proposed filter exhibits an excellent balance between noise suppression and edge enhancement (Fig. 10(c)). Unlike the VM filter, the proposed filter greatly enhances edges of various orientation and width and produces visually pleasing images.

To confirm or disprove the authors’ opinion on the filtering performance, a number of observers evaluated the quality of the restored artwork images shown in Fig. 10 with respect to the image sharpness and preserved edges, and the presence of residual noise. The choice of these criteria follows the well-known fact that a good filtering method should maintain or even enhance the structural content (edges, fine details) while it removes image noise. Original images and filtered outputs shown in Fig. 10 were viewed simultaneously under identical viewing conditions on the same LCD screen by 25 observers (21 male, 4 female; age range 22 to 40), unaware of the specifics of the experiment. Subjective evaluation was performed in a controlled room (external light had no influence on image perception) with gray painted walls using a calibrated, high quality displaying device with controlled illumination. One session was organized to complete the score test by all the observers who evaluated the restored image quality by picking the score from the following range: excellent (5), very good (4), good (3), fair (2), poor (1). Table 2 summarizes the subjective evaluation results. According to the average score, the images obtained using the sharpening VM filter were considered significantly better compared to results obtained using the standard VM filter. However, as the corresponding standard deviation values indicate, the observers were more unified in the evaluation of the images restored using the standard VM filter. This suggests that the evaluation of filtered images is rather difficult and very subjective task.

3.3. Discussion

In summary, as the filter analysis indicated, the amount of edge enhancement seen in Figs. 5, 6 and 10 increased by reducing value of $\alpha$ closer to $\alpha = 2$ (maximum enhancement) ranging filtering operations in (1) from refined ($\alpha = 8, 7$), trough modest ($\alpha = 6, 5, 4$) to extensive ($\alpha = 2, 3$) image enhancement. The above setting can be used to assist the user enhancing the images if the manual setting of $\alpha$ is required in the sharpening process based on the $3 \times 3$ supporting window. If the manual adjustment of $\alpha$ is not allowed due to the application constraints or is difficult, for example, due to the change of the processing window and/or the lack of prior knowledge about the processing solution, then the adaptive setting of $\alpha$ using (2) is recommended. It was demonstrated in Figs. 5–10 that the proposed adaptive VM sharpener can achieve the desired performance.
Fig. 7. Performance evaluation of the VM filter and the proposed sharpening VM filter with $\alpha$ by (2) using the noisy test images Fruit (left column) and Parrots (right column).
However, the above results do not demonstrate whether the proposed adaptive filter can successfully be used in conjunction with different supporting windows or to process spatiotemporal vector-valued signals. For instance, the spatiotemporal nature of color video or color image sequences...
suggests possible use of various temporal, spatial, and spatiotemporal supporting windows in the proposed framework. Such windows include a $3^3$ cube spatiotemporal window with $\zeta = \{(r + \phi, s + \varphi, t + 0); -1 \leq \phi \leq 1, -1 \leq \varphi \leq 1, -1 \leq t \leq 1\}$, an 11-point spatiotemporal window with $\zeta = \{(r, s, t - 1), (r + \phi, s + \varphi, t), (r, s, t + 1); -1 \leq \phi \leq 1, -1 \leq \varphi \leq 1\}$, a $3 \times 3$ square spatial window with $\zeta = \{(r + \phi, s + \varphi, t); -1 \leq \phi \leq 1, -1 \leq \varphi \leq 1\}$, and a 3-point temporal window with $\zeta = \{(r, s, t - 1), (r, s, t), (r, s, t + 1)\}$.

In the above notation, $r$ and $s$
indicate the spatial location whereas \( t \) indicates the temporal location (frame index). Fig. 11 and Table 3 summarize the results obtained using the two filters (VM filter and adaptive sharpening VM filter) and four windows under consideration. Both filters produced the best color video filtering results using the 11-point spatiotemporal window. The results also suggest that for sufficiently large windows (consisting of five and more pixels), the proposed filter usually achieves the desired performance and outperforms the VM filter, both objectively and subjectively.

### 4. Computational complexity analysis

Apart from the numerical behavior (actual performance) of any algorithm, its computational complexity is a realistic measure of its practicality and usefulness. Therefore, the VM filter and the sharpening VM filter are analyzed here in terms of normalized operations, such as additions (ADDS), subtractions (SUBs), multiplications (MULTs), square roots (SQRTs), and comparisons (COMPs).

Each one of the Euclidean distances \( d_{i,j}^{gh} \) requires two ADDs, three SUBs, three MULTs and one
SQRT for its evaluation. The number of unique $d_{ij}^{qh}$ calculations in the area of support $\zeta$ is equal to $O_E = \sum_{m=-1}^{\left| \zeta \right|-1} m$. Calculating the aggregated distances in (4) requires additional \(|\zeta|(|\zeta| - 2)|\) ADDs whereas the determination of the vector associated with the minimum aggregated distance requires \(|\zeta| - 1|\) COMPs. Thus, the total number of operations needed to calculate the output of the VM filter in (4) is: \([2O_E + |\zeta|(|\zeta| - 2)|\) ADDs + \([3O_E]\) SUBs + \([3O_E]\) MULTs + \([O_E]\) SQR Ts + \(||\zeta| - 1|\) COMPs. Alternatively, the objective function values in (1) can be calculated by subtracting \(|\zeta| - x|\) largest members of (3) from the aggregated distances in (4) to reduce the number of complex ordering operations for large $x$. Therefore, the cost of (1) for $1 < x < |\zeta|$ can be also expressed as follows: \([2O_E + |\zeta|(|\zeta| - 2)|\) ADDs + \([3O_E + |\zeta|(|\zeta| - x)|\) SUBs + \([3O_E]\) MULTs + \([O_E]\) SQR Ts + \([O_C + |\zeta| - 1|\) COMPs, where $O_C = |\zeta|\sum_{m=-1}^{\left| \zeta \right|-2} m$.

Table 4 summarizes the total number of operations for standard $3 \times 3$ window. Since the sharpening VM filter generalizes the VM filter and also provides additional functionality (i.e., edge enhancement), it was expected that the overall cost of the proposed filter exceeds the cost of the standard VM filter.

The execution of the color image filtering tools, on an Intel Pentium IV 3.00 GHz CPU, 2.00 GB RAM box with Windows XP operating system and MATLAB 7.0 programming environment, took (on average) 10.647 and 15.578 s per $512 \times 512$ image employing the VM filter and the sharpening VM filter with $x$ obtained using (2), respectively. Note that the software implementations have not been optimized and that the development of a fast algorithm of the proposed filter is beyond the scope of this work.

5. Conclusion

The sharpening vector median (VM) filter was introduced. Unlike previously proposed vector filters, the new filter can simultaneously perform noise smoothing and edge enhancement by trimming the aggregated distances used in the constrained minimization criterion employed by the operator. The new filter was shown to exhibit essential filtering properties, including root signals, zero impulse response, and invariancy to rotation, scale and bias. An application to filtering of color images was shown and the sharpening VM filter was successfully tested using well-known color images and video with synthetic noise as well as digitized artwork images with real noise.

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