Location of Mobile Terminals using Time Measurements and Survey Points

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Abstract—**The location of mobile terminals has recently become a topic of much research. One of the more popular methods of radio location is the Time Difference of Arrival (TDoA) method where the location of the mobile terminal is estimated using measurements of the differences in propagation times from the mobile terminal to three or more base stations. The traditional approach used to perform the location estimation is the least squares algorithm assuming zero mean Gaussian measurement error distributions. In the urban microcell radio propagation environment with Non Line of Sight (NLOS) propagation, this assumption is often violated causing large location estimation errors. This paper demonstrates an estimation algorithm based on non-parametric density approximation of the measurements using a survey of time difference measurements. This method gives high accuracy location estimates even in the presence of NLOS propagation.**

I. Introduction

There is much interest within the wireless communications research community on technologies that can estimate the location of mobile terminals. Mobile terminal location technology can be used for finding callers making E911 calls, location sensitive browsing, and assisting with resource allocation [1], [2], [3], [4].

Several methods have been proposed to locate wireless mobile terminals based on measuring the Angle of Arrival (AoA) of the radio signal at base stations, measuring the Received Signal Strength (RSS), measuring the Time of Arrival (ToA), and measuring the Time Difference of Arrival (TDoA) of the radio signals. The channelization schemes proposed for next generation cellular networks such as CDMA make TDoA location scheme most attractive because the multiple access schemes allow for high accuracy time measurements[5]. This paper will therefore concentrate on location based on TDoA measurements.

The TDoA location technique involves measuring the differences between the times it takes the mobile terminal's signal to travel from the mobile terminal to the measuring base stations.

The difference between the propagation times from the mobile terminal for pairs of measuring base stations is measured. Each time difference measurement defines in the case of LOS propagation and error free measurements a hyperbolic line upon which the mobile terminal must reside. Two difference measurements, derived from time measurements from three base stations, allow the mobile terminal position to be calculated[6].

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Complications arise from two sources. First, the measurements in the field are not error-free so that estimation techniques must take into account measurement noise. Second, the propagation in some cases is Non-Line of Sight (NLOS) when the LOS path is obstructed by a building or geographic feature. In the NLOS case, the delay measurements will be biased from the LOS values and the location techniques described above will lose accuracy or in some cases return ambiguous results.

If the joint density of the measurement vector of propagation delays and location of the mobile terminal is known, then given a measurement of delays for a mobile terminal it is possible to calculate an optimal estimate of the mobile terminals position[7]. The difficulty is that the joint density is not known.

This paper discusses a technique of estimating the conditional density of the measurements given the mobile terminal location using a survey of propagation delays measured at known locations.

Section 2 describes the estimation technique. Section 3 describes the simulations used to evaluate the locations estimate technique. Section 4 contains the results of the simulation studies. Section 5 summarizes our conclusions.

II. Estimation Technique

This paper concentrates on 2-D location estimation since this is the case of greatest interest for wireless networks. The methods can be scaled up to higher dimensions easily. Thus, the location vector is $\boldsymbol{\theta} = (x, y)$ where x is the x coordinate and y is the y coordinate.

Propagation time measurements are converted to propagation distance measurements by multiplication by c, speed of light. The measurement vector is denoted as $\boldsymbol{Z}$. We will define the measured propagation distance difference vector as

$$[\boldsymbol{Z}]_j = (d_j(\boldsymbol{\theta}) - d_0(\boldsymbol{\theta})) + V_j. \tag{1}$$

where

$$d_j(\boldsymbol{\theta}) = \sqrt{(x - x_j)^2 + (y - y_j)^2} \tag{2}$$

with (x_j, y_j) being the location of the j^{th} base station and V representing measurement noise. Base station 0 provides the time reference against which all other time propagation measurements are made.

Most work on location estimation methods is based on the Maximum Likelihood Estimator (MLE) technique. The true

MLE cannot be used because the conditional probability density is unknown. Instead a MLE based on an estimated density is used. The resulting Approximate MLE (AMLE) estimate, θ_{AMLE}, can be written as

$$\hat{\theta}_{AMLE} = \arg(\theta) \ \max \ \hat{f}_Z(Z|\theta; P), \quad (3)$$

where $\hat{f}_Z(\cdot)$ is the estimated density, and P is the assumed propagation model. The accuracy of the location estimates is dependent on the accuracy of the estimated density and assumed propagation model. In the literature, the propagation model used is LOS propagation and the measurement noise vector is assumed to be jointly Gaussian.

In practice, a cellular network has some knowledge of the location of the mobile terminal. The hand off algorithm determines which base station serves the mobile terminal at any given time. Knowing which base station is serving a mobile terminal gives statistical knowledge about the mobile terminal location. This knowledge can be used to improve the accuracy of location estimates.

If prior information exists, the criterion of optimality changes and MLE estimation is no longer optimal. In terms of Mean Square Error (MSE), defined as $\mathrm{E}\{[\hat{\theta}-\theta]^T[\hat{\theta}-\theta]\}$ (superscript T designates matrix transpose), the optimal estimator is the Minimum Mean Square Error (MMSE) estimator which is given by

$$\begin{aligned} \hat{\theta}_{MMSE} &= \mathrm{E}[\theta|Z] \\ &= \int_S \theta f_\Theta(\theta|Z)\, d\theta \end{aligned} \quad (4)$$

where $\mathrm{E}[\cdot]$ denotes the expectation operator, $f_\theta(\theta|Z)$ is the conditional probability density of location, θ, given the measurement vector, Z, and S is the region the mobile terminal is known to reside in[7].

Unfortunately, as for the MLE case above, the densities in (4) are not known in real world implementations so we must use the Approximate MMSE (AMMSE) estimator, given by

$$\hat{\theta}_{AMMSE} == \frac{\int_S \theta \hat{f}_Z(Z,\theta)\, d\theta}{\int_S \hat{f}_Z(Z,\theta)\, d\theta}, \quad (5)$$

where $\hat{f}(\cdot)$ designates an estimated density function. There are two methods of estimating the density functions: parametric and non-parametric.

The parametric technique calculates the estimated density functions assuming a model for the noise and a propagation model. A few parameters for the models are derived from field measurements such as variances of noise. The difficulty in location estimation is that simple parametric models of the propagation environments only exist except for LOS propagation cases[8]. Dealing with NLOS propagation cases, where the straight line path is blocked by a building or geographic feature is much more complex [9], [10], [11].

The non-parametric technique estimates the densities using survey data taken from the environment with pre-assuming any models. This survey data can be obtained from field measurements or generated using computer ray-tracing propagation models. Data sets such as this are already generated to evaluate base station location and network performance.

Kernel name	Kernel function $K(x)$
Parzen Laplace[16]	$\frac{1}{2}\exp\left(-\|x\|^1\right)$
Generalized Gaussian[13]	$K_g\|C\|^{-\frac{1}{2}}\exp\left(-\frac{x^TC^{-1}x}{2}\right)$
Distance based[17]	$\prod_{j=1}^{k}\frac{1}{\pi}\frac{1}{1+(x_j)^2}$

$\|x\|^p$ is the L_p distance of x from the origin.

$K_g = (2\pi)^{-\frac{k}{2}}$.

TABLE I

KERNEL FUNCTIONS

The estimation problem is restated as estimating the location of a mobile terminal at an unknown location within region S from measurement vector Z given the survey set. The survey data set consists of measurements $\{Z_1, Z_2, ..., Z_n\}$ made at respective true locations $\{\theta_1, \theta_2, ..., \theta_n\}$.

First we pick kernel functions $K_z(z)$ for the measurements and $K_\theta(\theta)$ for the locations. The joint density of locations and measurements is approximated as a sum of the product of the kernel functions [12]:

$$\hat{f}_{\Theta,Z}(\theta, Z) = \frac{1}{n}\sum_{j=1}^{n}(h_z)^k(h_\theta)^2 K_z\left(\frac{Z-Z_j}{h_z}\right)K_\theta\left(\frac{\theta-\theta_j}{h_\theta}\right), \quad (6)$$

The constant k is the number of base stations used to locate the mobile terminal. The smoothing constants h_z and h_θ determine the width of each of the kernel functions. For simplicity, one usually chooses kernel functions with the properties[13]:

(a) $K(w) \geq 0 \, \forall w \in \mathcal{R}^k$

(b) $\int_{\mathcal{R}^k} K(w)dw = 1$

(c) $\int_{\mathcal{R}^k} wK(w)dw = 0$

where k is the dimension of the kernel. The kernel functions with properties given above are k-variate density functions for random variables with zero mean vectors.

If we substitute the estimated density from (6) into the AMMSE equation, (5), and the kernel functions satisfy properties (a), (b) and (c) above then the resulting estimator is given by

$$\hat{\theta}_{AMMSE} = \frac{\sum_{j=1}^{n}\theta_j(h_z)^{-k}K_z\left(\frac{Z-Z_j}{h_z}\right)}{\sum_{j=1}^{n}(h_z)^{-k}K_z\left(\frac{Z-Z_j}{h_z}\right)}. \quad (7)$$

This estimated position is a weighted averaged of the survey point positions with the weights being determined by the measurement data[12].

The kernel functions used in this paper are listed in Table I. These kernel functions are selected because of good performance when applied to estimation in similar problem domains[12], [13].

The Generalized Gaussian is used for TDoA location because there will be a non-zero correlation between elements in the measurement vector due to the subtraction of the refer-

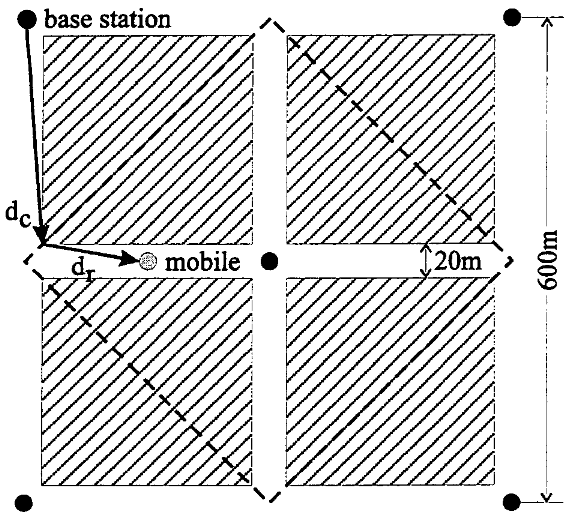

Fig. 1. Propagation Environment

ence base station distance measurement[6]. The C matrix is the covariance matrix for the measurement vector V.

Selecting n and h_z is a difficult task, made more difficult since the optimal values of each is dependent on the value of the other. Larger values of h_z result in each survey point having a larger region of influence in the sample space for the estimated density. Small values of h_z mean that the influence region of each survey point is small with the estimated density function becoming a sum of delta functions as $h_z \rightarrow 0$.

The optimal values of h_z and n for each kernel are a function of the actual density function being estimated and are thus unknown for any given estimation problem. It is, however, known that using values of h_z and n that have the correct order of magnitude allows one to obtain results almost as good as using those obtained using the optimal value in many cases[13].

The value of n used depends on the accuracy desired. The distance between the survey points should be of the same order of magnitude to the distance error tolerated. Unfortunately, there is a minimum achievable distance error because of measurement noise[14].

An examination of the kernel functions in Table I and (6) reveals that in h_z has some relation to the standard deviation of the measurement noise. Therefore, it appears that h_z is of the same order of magnitude as the standard deviation of the measurement noise. In Section 3, the relative insensitivity of the estimation technique to variations in the value of h_z will be shown as long as h_z is of the correct order of magnitude will be shown.

III. Simulation Description

A square microcell with sides of length $300\sqrt{2}$ metres long was used to evaluate the estimator accuracy. The base station configuration is shown in Figure 1. The shaded regions represent buildings. This configuration was used since it has been used to evaluate other mobile terminal location schemes[15].

When the LOS path between a base station and mobile terminal is unobstructed, the propagation distance is simply the Euclidean distance between the mobile terminal and base station. For NLOS propagation, the radio signal is assumed to diffract around the corners of buildings. The propagation path is thus the distance from the base station to the corner plus the distance from the corner to the mobile terminal. This propagation distance is $d_c + d_r$ as shown in Figure 1.

Measurements from 3 base stations are used to locate the mobile terminal, $k = 3$. The measurement noise is modeled as a jointly Gaussian random variable with a covariance of

$$C = \mathrm{E}\left[VV^T\right] = \sigma^2 \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}.$$

The base station closest to the mobile terminal is the reference base station and the two other base stations with the lowest measured propagation distances are used. Others have assumed that the closest base stations are used but this policy was rejected as it gives the estimator information that would not be available in the field[15]. For the simulations the prior location density function is a uniform density function over the diamond shaped region in Figure 1. Only survey points where the 3 base stations selected for location were in the set of the 4 base stations with the lowest surveyed distance measurements are used to locate the mobile terminal. The figure of merit used to judge performance is the Root Mean Squared Error (RMSE).

The first set of simulations were performed with 100 survey points, $n = 100$. The value of h_z was varied to see how robust the estimators are to variations of this smoothing parameter.

The second set of simulations were performed to show the robustness of the kernel estimators to different sizes of the survey set. The number of survey points, n, was varied from 10 to 100 and the RMSE of the different estimators was recorded.

The third set of simulations was performed to measure the performance at different levels of measurement noise. The value of σ is varied from 15 metres to 50 metres and the RMSE recorded. The survey points are placed in two perpendicular lines down the center of the vertical and horizontal streets with regular spacing between points.

IV. Results

Figure 2 presents the error values for the kernel estimators when h_z was swept from 0.1σ to 3σ. The optimal value of $h_z \approx 2.0\sigma$ with the Generalized Gaussian kernel estimator giving the best results. The estimator is robust to variations of h_z. The distance-based kernel estimator does not work as well as the other kernel based estimators with higher RMSE and more sensitivity to variations of h_z.

The results for varying values of n are shown in Figure 3. Once the survey set reaches a certain size, new survey points are adding mostly redundant information. This point appears to be around $n = 40$ for this environment. The decision of how many survey points are needed is a trade off between the cost of taking the survey measurements and how much accuracy is desired.

The third set of simulations measured the performance of the estimators as the standard deviation of the measurement noise, σ, was varied. The results for simulations with random survey point locations are shown in Figure 4. The Approximate MLE and parametric MLE errors are also shown. The non-parametric estimators error was lower for all σ than the MLE errors. The parametric estimators' error curves are so close that they overlap on this graph.

V. Conclusions

This paper has introduced the use of non-parametric kernel-based estimators for location of mobile terminals in wireless networks using measurements of differences of propagation delays. It has been demonstrated that these estimators perform better than the previously used parametric MLE estimators for the case of a simulated micro-cell environment with LOS and NLOS radio propagation. These estimators were shown to give excellent results at several different levels of measurement noise. Methods for calculating good values for parameters of the kernel functions were demonstrated as well as the robustness of the estimators when the values of the parameters vary from the optimal points.

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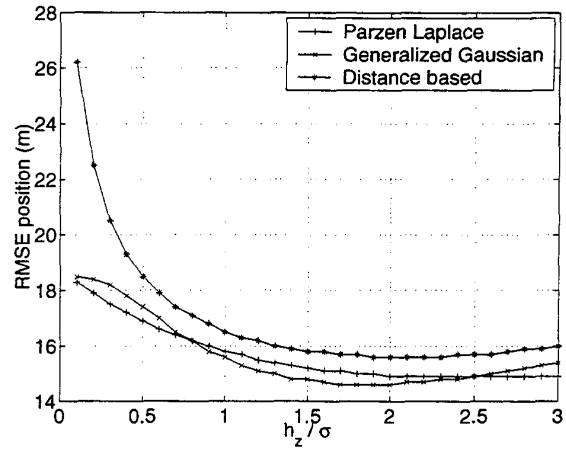

Fig. 2. TDoA Estimator performance for differing h_z values ($n = 100$, σ=15m)

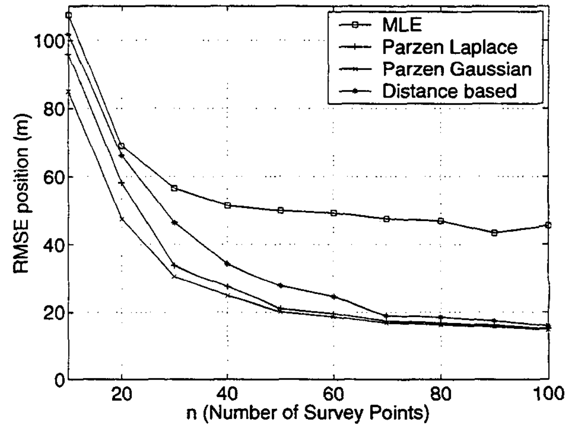

Fig. 3. TDoA Estimator performance for differing n values (σ=15m)

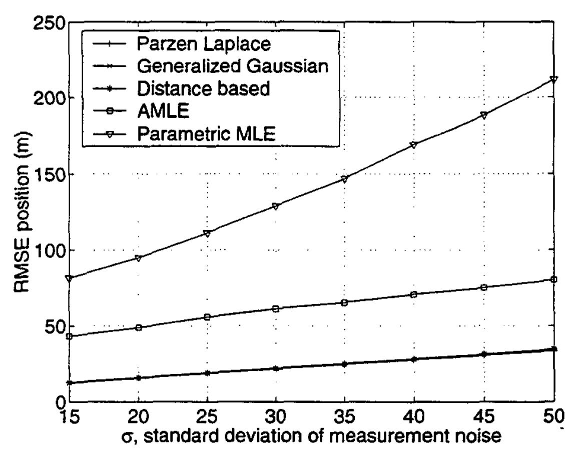

Fig. 4. Comparison of parametric ($n = 100$) and non-parametric estimators